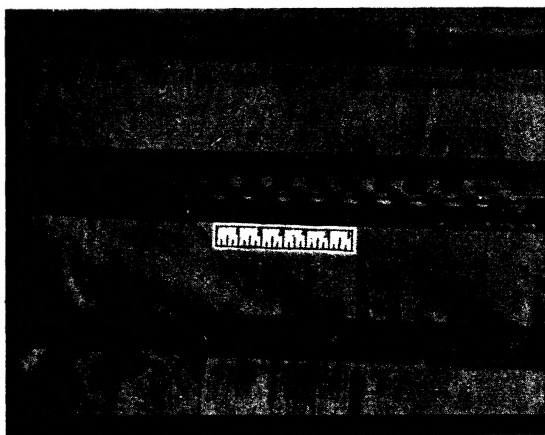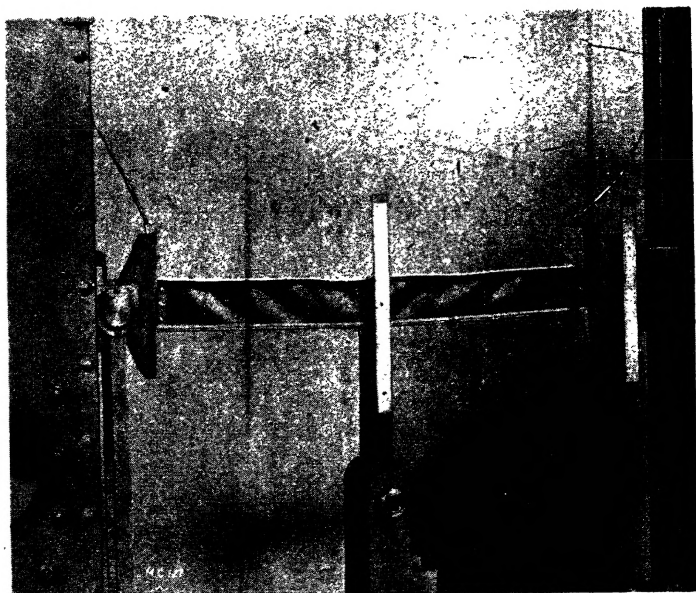**REQUEST**

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PROPERTIES AND
STRENGTH OF MATERIALS
(METALS)

COMPRESSION FAILURE OF A SPAR FLANGE BY ELASTIC INSTABILITY

WAGNER BEAM, SHOWING ELASTIC INSTABILITY OF WEB UNDER COMPRESSION

Frontispiece

An Introduction *to*

Aeronautical Engineering

FOR STUDENTS ENGAGED IN ALL BRANCHES OF
MECHANICAL ENGINEERING, WITH SPECIAL
REFERENCE TO AERONAUTICAL WORK

Vol. III

PROPERTIES AND
STRENGTH OF MATERIALS
(METALS)

BY

J. D. HADDON, B.Sc.

Fellow of the Royal Aeronautical Society
Author of " A Simple Study of Flight "

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1942

INTRODUCTORY

THIS series is intended for the use of all those who are engaged in practical aeronautical engineering, and who feel the need of at least an elementary knowledge of the theory underlying their practical work. For this reason it is hoped that it may appeal equally to draughtsmen, apprentices, pilots and students at technical schools who are desirous of entering some kind of aeronautical work.

The authors have aimed at producing books which, while avoiding the use of higher mathematics, are at the same time correct, up to date and as free as possible from the technical errors so often found in books which attempt to be " popular " at the expense of everything else. To what extent the authors have succeeded in their object the reader alone can judge.

Except in the methods of putting the subject before the reader, no claim to originality can be made in books of this kind, and the authors acknowledge their indebtedness to all the standard works on aeronautics.

The present series consists of—

Vol. I. Mechanics of Flight, by A. C. Kermode.

II. Structures, by J. D. Haddon.

III. Properties and Strength of Materials, by J. D. Haddon.

Vol. I outlines the principles which maintain an aeroplane in flight, Vol. II is confined more to the internal structural problems of aeroplane design, while Vol. III deals with the strength of the members and the materials used in them.

Subsequent volumes will deal with other subjects directly connected with aeronautical work.

The authors will welcome criticism of any kind.

A. C. K.

J. D. H.

PREFACE TO THIS VOLUME

THIS volume deals with the principal metals used in the structure of aircraft, and the methods of determining the sizes of the members from the loads in them. The methods of determining the loads are given in Vol. II of the series.

I have endeavoured to leave out those details which are usually confusing to the beginner and are only of interest to the specialist. At the same time it is hoped that sufficient explanations and exercises are given, so that the subject may be learnt by being understood, rather than by memory of formulæ.

Although the book is primarily written for the aeronautical student, it should be of equal value to students of any branch of mechanical engineering.

As with the two companion volumes, I have assumed that the reader is not familiar with higher mathematics. He is, however, strongly advised to study the calculus if he intends this book to be an introduction to more advanced work. The work on continuous beams may be found a little advanced by some readers, but has been included to meet the request of a reviewer of Vol. II.

In a book of this kind I am obviously indebted to the work of previous writers on the subject. Also I would particularly like to thank Dr. W. H. Hatfield, of the Brown-Firth Research Laboratories, for providing me with prints for Figs. 3, 4, 6, 7, 17, 19, 20, 23, 24, 25 and 26 and for very valuable help in suggesting improvements in the first four chapters; the Bristol Aeroplane Company for providing the frontispiece, and Figs. 142 and 143; and Captain W. Laidler for checking the numerical work.

PREFACE TO FOURTH EDITION

SINCE the first edition several alterations and additions have been made the more important being: Nitriding, normal stress, torsion test, principal stresses, Wagner beam and elastic instability of struts.

I would like to thank those who have given constructive criticism of past editions.

<div align="right">J. D. H.</div>

CONTENTS

ix

STRUCTURE OF PURE METALS AND ALLOYS

The Structure of Matter.

. We cannot thoroughly understand the materials we use unless we have some knowledge of their internal structure. The object of this chapter is to as briefly as possible supply that information.

All materials, whether solid, liquid or gaseous, are composed of minute structural units called molecules, which are so small that they are not recognizable under the strongest microscope. These molecules are so arranged as to leave gaps (not of air, but of space) between them, which vary in extent with the variation of pressure and temperature.

The molecules, which vary in composition for different substances, are not solid particles of matter, but are made up of one or more atoms. The atoms are the same for any one elementary substance, but vary both in their mass and properties for different substances.

In an elementary substance such as hydrogen, carbon, aluminium, etc., the molecules are composed of atoms of the same kind, and in a few—*e.g.*, helium and iron—the molecule contains only one atom. In chemical compounds—*i.e.*, substances made up of two or more elementary substances combined together—the molecules are formed by the close union of the atoms of the different elements of which the substance is composed—*e.g.*, water, the best known of all chemical compounds, is made up of two parts of hydrogen to one of oxygen, and its molecule contains two hydrogen atoms and one oxygen atom.

The atom is composed of a nucleus which is a positive charge of electricity, around which rotates one or more negative charges, called electrons.

The radius of an atom is in the order of 10^{-8} cm., of an electron $1 \cdot 9 \times 10^{-13}$ cm., and a nucleus considerably less ; that of the hydrogen nucleus is 10^{-16} cm. This may be illustrated by imagining an atom magnified to the size of the earth ; then an electron would correspond to the dome of St. Paul's on the surface and the nucleus to a football at the earth's centre. All the rest would be space.

It should be clear from the foregoing that a substance, whether solid, liquid or gas, is practically all space. Even its mass is determined by the positive charge on the nuclei of the atoms of which it is composed.

The molecules tend to cling to one another, in the same way that magnets do when opposite poles are presented to each other or opposite charges of electricity. In fact, some such magnetic and electric forces are at work.

These cohesive forces which exist between molecules decrease in magnitude as the distance between them is increased, somewhat in the same way that the force of gravity decreases as we move further above the earth's surface.

The difference between the solid, liquid and gaseous state of a substance depends on the forces between the molecules which comprise it.

A solid may be said to be made up of molecules, each of which vibrates in a small space, but their cohesive force is such that in ordinary circumstances they cannot leave their relative mean positions.

The vibration of the molecules is heat. As a solid is heated, this vibration increases ; and in the same way that a fighting crowd requires

more room than one that is calm, so do the molecules require more room with increased vibration. Thus a substance expands on heating, and also, the distance between molecules becoming relatively greater, they lose some of the force which exists between them.

This force therefore decreases with increase of temperature until it is no longer sufficient to preserve the relative positions of the molecules. This is where melting takes place and the substance becomes a liquid.

A liquid may therefore be said to be a substance in which the molecules are free to move relative to each other, but their vibration and cohesion is such that their average distances apart are maintained.

Evidence of molecular motion in a liquid may be obtained by placing a grain of indigo in an outwardly still liquid. The colour will very quickly spread to the whole liquid.

The large pressure required to compress a liquid the smallest amount shows that the molecules maintain their average distances apart.

Evaporation from the surface of a liquid takes place due to molecules reaching the surface with a relatively high speed of motion, such that their momentum carries them to such a distance above the surface that the mutual attraction of the molecules is lost and they become free to mingle with the gas over the liquid and behave as other gas molecules. This process may be hastened by the application of heat, which increases the speed, and so the power of the molecules to " jump out " of the liquid.

Gases need not be considered in this book : briefly they are substances in which the molecules are so far apart that their cohesive properties are lost, and they thus travel in rapid haphazard motion.

Molecules of different substances differ both in form and their cohesive forces. When the forces are weak, very little movement— *i.e.*, heat—is required to change the substances from their solid state into liquids or gases, and the melting-point will be low.

For instance, the attractive forces of the diamond molecules are very great, and it would need a very, very high temperature to change it into a liquid, while with such substances as oxygen the forces are so weak that an extraordinarily low temperature is required to solidify it.

Crystals.—When a substance solidifies, the molecules join together as if there were definite points of attachment on each which must be brought together before full cohesion takes place.

As a liquid cools and the heat motions decrease, the molecules begin to attach themselves together at a number of points in the liquid, definitely arranging themselves and cohering in a preferred orientation and arrangement.

As might be expected from this orderly building up of a solid, the solid itself will have a regular form unless the building has been interfered with. A solid thus formed is called a crystal.

Thus as a liquid cools, molecule after molecule will take its place in orderly array with those already joined together and minute crystals are formed. These rapidly grow in size by throwing out arms, called dendrites, into the surrounding liquid ; these arms throw out other arms and these still smaller arms, until the solid crystal is formed.

Everybody is familiar with the six-pointed snow-flake which may be observed in a snowstorm on a very cold day. These are examples of dendritic crystallization which have been arrested in the process for want of further material. When water freezes, the gaps fill up and a hexagonal prism is formed. If a sheet of clear ice is placed in a lantern and focused on a screen, the heat will unbuild the ice crystals in the inverse order in which they were built and six-pointed cavities will appear, looking like snow-flakes on the screen.

Fig. 1 (*a*) shows an ice dendrite, or snow-flake, in an early stage, and (*b*) one in a later stage of formation ; (*c*) shows an iron dendrite, and here it will be noticed the arms are joined at right-angles.

FIG. 1

It may be asked why we do not find more bodies in crystalline form. The reason is that, although many of the things in daily use are crystalline, the fact is not apparent, due to the crystals being either too small or, their growth having been interfered with by adjoining crystals, their regular form is not noticeable.

In order that a liquid on cooling may form into a perfect crystal it must grow from a single point. If there are many points where the molecules start to join, there will be many crystals, which will grow with a regular form until they are arrested by neighbouring crystals, after which they will only be able to grow in that direction not yet occupied by other crystals. The result will be a substance made up of a mass of small irregular crystals, which are too small to distinguish with the naked eye. To grow large, perfect crystals it must be arranged that the molecules find few centres from which to grow, and the cooling must be very slow, so as to give plenty of time for each molecule to settle itself in its correct place.

An important point to be remembered is that the quicker the cooling, the smaller the crystals.

Cleavage Planes.—Another important fact is that, due to the arrangement of the molecules, there are certain parallel planes in a crystal, called cleavage planes, along which the crystal is weaker than in any other direction. If one tries to break a crystal, it will fail along the cleavage planes and act in much the same way as wet photographic plates will when tried to be parted. This is illustrated in Fig. 2.
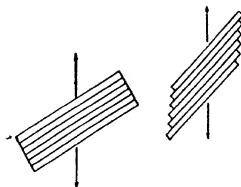

FIG. 2

When a piece of pure metal is broken, the crystals are usually broken across at the cleavage planes and not pulled apart, thus showing that the cohesion between the surfaces of adjoining crystals is stronger than that within the crystal.

The Structure of Pure Metals.

Pure metals in the cast or annealed state always present under the microscope a typical appearance. This appearance is shown in Figs. 3 and 4 for specimens of pure iron ; they might do equally well for any
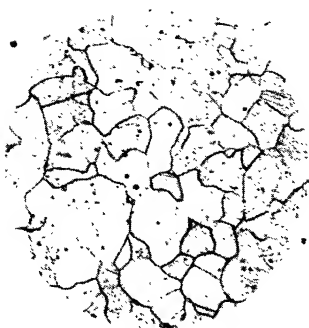
FIG. 3

FIG. 4

other pure metal, for the similarity is such that they are indistinguishable under the microscope.

As the figure shows, the structure is made up of a number of rough polygonal grains. These grains are true crystals, the growth of which has been interfered with by neighbouring crystals, thus their lack of external geometrical symmetry. As previously mentioned, the character of crystals does not depend on their geometrical form, but on the orderly arrangement or orientation of the molecules of which they are formed. The outline is only the result of this internal arrangement, and is only regular so long as it is not interfered with externally.

It will be noticed that although the crystals in Fig. 4 are identically constituted they present a different appearance under the microscope, some showing darker than others. This is due to the way in which specimens are prepared. They are first polished and then, in order to reveal the structure more clearly, they are etched—i.e., the surface is slightly dissolved by the application of a suitable solvent. It must be remembered that a crystal is built up of a number of separate orderly arrangements of the molecules, which first form dendrites. When the surface is dissolved by the etching reagent, the crystal surface dissolves preferentially in certain portions. In some cases the arms of the dendrites will reappear in the same way that the "snow-flowers" appear on melting ice. More usually the surface will

FIG. 5

have a number of facets all facing the same way for any one crystal but differing in direction for different crystals. This is illustrated diagrammatically in Fig. 5. The light striking these facets will be reflected in one direction for any given crystal ; those which reflect light back into the microscope appearing light, whilst those which reflect it at an angle appear darker.

Fig. 3 does not show this difference in shading as it has not been etched so deeply. Neighbouring crystals may etch at different rates according to their orientation, thus creating differences of level which reveal " steps " at the crystal boundaries.

Solid Solutions.—Before going further the meaning of the term " solid solution " should be explained.

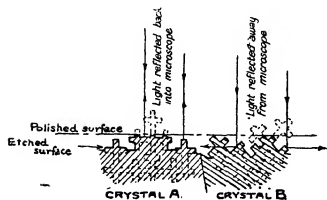An ordinary liquid solution is one in which there is a complete merging of the constituents, such that their separate existence cannot be detected under the highest-powered microscope. Also this merging must remain complete for different proportions of the constituents. In a solution the molecules of the different constituents are mixed together, and not combined to form different molecules as in a chemical compound.

Some substances, including many metals, in passing from the liquid to the solid state retain the cha racteristics of liquid solutions—*i.e.* their constituents remain completely merged and in indefinite proportions. Such substances when in the solid state are called solid solutions.

A liquid solution may become saturated and be unable to hold more of a constituent. It is usual for a cold liquid to become saturated sooner than a hot one. Thus if a hot solution of, say, soda and water, near the saturation point, is allowed to cool, it will become saturated, and as it cools still further the soda will be in excess of the limit of solubility and some of it will separate out in the form of pure soda crystals.

In the same way a solution of two metals in a solid state may on cooling become saturated, in which case there will be, or tend to be, a separation of the excess metal, which will form pure metal crystals, and the remainder solid solution.

Structure of Alloys.

When two or more molten metals are mixed together and allowed to solidify, the resulting solid is called an alloy.

Such a metallic alloy has three modes of solidification—

(1) It may form a solid solution.
(2) It may separate during solidification and form crystals of pure metals.
(3) It may form a mixture of pure metal crystals and solid solution, or mixtures of different solid solutions each having its own composition and characteristics.

Alloys of the first group retain their state of solution undisturbed by the process of solidification, and the crystals thus formed each contain the same composition as the molten liquid. The micro-structure of such a metal will be similar to that of a pure metal.

In the second group the constituents making up the liquid solution are separated out during the process of solidification, and the solid alloy becomes a mixture of crystals of pure metals.

Alloys which crystallize as those in the third group are the most numerous. If the two constituent metals are present in such proportions that the second metal is in excess of its limit of solubility in the first metal when they are in the solid state, there will be a separation of the excess metal on solidification. The resultant solid will be made up of crystals of the excess metal together with a saturated solid solution of the two metals.

Eutectic.

The word " eutectic " is used to indicate the mixture or alloy possessing the lowest freezing-point which can be prepared out of the two

metals in question ; it corresponds with a definite proportion of the metals, has a fixed freezing-point, and forms a finely divided aggregate of the two constituents.

Take, for example, alloys of the metals A and B, which form a eutectic on freezing, and let the proportions of the metals forming the eutectic be 40 per cent. A, and 60 per cent. B.

If there is more than 40 per cent. A present in the cooling liquid alloy, on reaching a temperature which will be lower than the freezing-point of pure metal A, solidification will commence by the separation of crystals of pure metal A. As the temperature is further lowered, more of these pure metal crystals will separate, thus making the remaining liquid weaker in A. When the eutectic proportion is reached—*i.e.*, 40 per cent. of A, 60 per cent. of B—the remaining solid will freeze at a constant temperature, and form pure crystals of the two metals deposited side by side, called the eutectic alloy of A and B. The resulting solid will be comprised of pure metal A and eutectic.

If the original mixture contains 40 per cent. of A and 60 per cent. of B, the whole alloy will form eutectic, and freeze at one temperature, this temperature being the same as that at which the eutectic was formed in the previous case.

If there is less than 40 per cent. of A in the cooling alloy, freezing will commence at a temperature below that of pure metal B, by the separation of crystals of pure metal B. As the mixture is further cooled more of the metal B will separate out, thus making the remaining liquid richer in A, until the eutectic proportion is reached, when this remaining liquid will form the eutectic at the same constant temperature as before. The resulting solid will be comprised of pure metal B and eutectic.

It follows that all the alloys of A and B will form eutectic at the temperature at which the 40 per cent. of A, 60 per cent. of B alloy freezes, which is the lowest freezing-point of the series. Also if the alloy originally contained more of the metal A than the eutectic composition, the solid will contain crystals of A and eutectic, and if more of the metal B, it will contain crystals of B and eutectic.

The structure of eutectiferous alloys which have one metal in excess of the proportion required to form the eutectic consists of crystals of the excess component embedded in the eutectic, or as in the case of iron alloys when the eutectic is surrounded by the excess constituent.

Figs. 6 and 7 show the iron-carbon eutectic as represented in " white iron " containing 3·8 per cent. carbon, at 60 and 300 magnifications respectively.
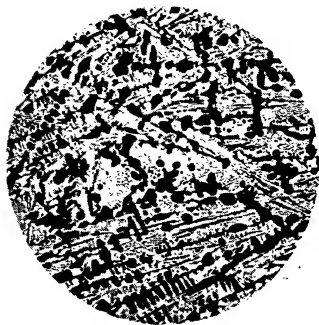

FIG. 6

FIG. 7

SOLIDIFICATION OF PURE METALS AND ALLOYS

Cooling Curves.

The method of alloying two metals is to melt the metals together and allow them to solidify. The study of the phenomena which prevail when metallic bodies are cooled from temperatures where they are completely fluid down to air temperature gives a guide to the constitution of the metallic mixtures.

If, when a metal is cooled slowly from the molten state, readings of temperature and time are taken, and these are plotted as ordinates and abscissæ to form a graph, a diagram, called the cooling curve of the metal, is obtained.

A typical cooling curve for a pure metal is shown in Fig. 8. This shows one arrest in the fall of temperature at *ab*, and this is the temperature where freezing occurs ; it being a well-known fact that during freezing and melting the temperature remains constant, the heat lost from or given to the body during that time being the latent heat of fusion.

Another type of cooling curve, which has the advantage of showing minute temperature changes more clearly, is shown in Fig. 9. In this type the temperature is plotted against intervals of time taken by the metal to fall through successive equal differences of temperature.
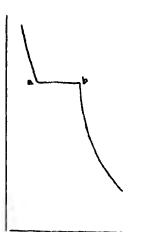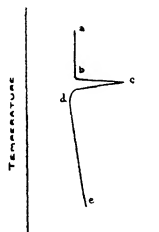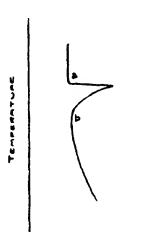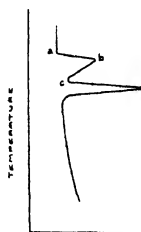
| FIG. 8 | FIG. 9 | FIG. 10 | FIG. 11 |

If the metal cools at a uniform rate the curve is a vertical straight line as *ab*, Fig. 9. An arrest in the temperature fall is shown by the point *c*, whilst the curve outwards *de* shows a gradual decrease in the rate of cooling.

The cooling curve for an alloy will be different to that for a pure metal. First take the case of an alloy of two metals forming a solid solution ; it has already been shown that such an alloy forms a homogeneous mass of crystals similar to a pure metal, and it might well be thought that the cooling curve would be the same as that of a pure metal. There is, however, this difference—that whereas in a pure metal freezing takes place at one temperature, called the freezing-point, with a solid solution the freezing extends over a range of temperature. A cooling curve for such an alloy is shown in Fig. 10. The commencement of freezing takes place at *a*, and is well marked. The alloy has not completely solidified until the temperature *b* is reached and the curve has returned to its normal position. *ab* is known as the cooling range of the alloy.

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It is usual for an alloy to commence to solidify at a lower temperature than the freezing-point of its principal constituent, and the greater the proportion of the second constituent the lower the temperature.

When a liquid that forms a solid solution freezes, the solid which separates first forms homogeneous crystals of the two constituents containing a larger proportion of the principal constituent than the solution as a whole. Hence the remainder will be richer in the second constituent and have a lower freezing-point. The freezing-point is in this way gradually lowered (from *a* to *b* on the cooling curve) as the remaining liquid becomes richer in the second constituent until solidification is complete.

If the rate of cooling is too quick, the crystals formed will contain a core of metal relatively poor in the second constituent and getting richer from the centre outwards. With slow cooling, however, diffusion of the second constituent takes place from the outside to the already solid parts which are weak in this constituent, thús forming a homogeneous alloy.

In the case of a eutectiferous alloy, solidification begins by the crystallization of a small quantity of the principal constituent ; this is marked by a small arrest in temperature as shown by the line *ab* in the cooling curve for such an alloy (Fig. 11). The remaining liquid will now be richer in the second constituent, hence the temperature at which further quantities of the principal constituent are deposited will be lower and the cooling rate still retarded as shown by the line *bc*. When the temperature *c* is reached, the solution has become saturated in the second constituent and hence any further deposit of the principal constituent necessitates a deposit of the second. At this temperature the remainder of the liquid will freeze by forming a eutectic.

Constitutional Diagrams.

By obtaining the cooling curves of a number of alloys containing the same two metals in varying proportions, a new diagram may be obtained, called a Constitutional or Equilibrium Diagram.

To construct such a diagram, the percentage composition of a series of alloys is plotted against the temperature at which an arrest is shown in the cooling curve. Fig. 12 illustrates the construction of such a diagram.
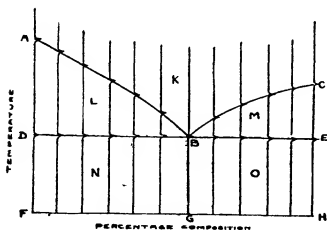
FIG. 12
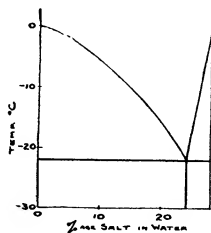
FIG. 13

The line *ABC* shows the different temperatures at which solidification commences for varying proportions of the two metals. At any point *K* above *ABC* the mixture is liquid. The line *DBE* represents the final freezing-point—*i.e.*, the temperature at which the alloy becomes completely solid and the eutectic is formed. It will be noticed that the temperature is the same for all compositions. The point *B*, where there is only one arrest in the cooling, shows the composition at which the alloy will be composed completely of eutectic.

Let the two metals forming the alloy be called X and Y. Any point L within the area ABD will represent an alloy partly solid and partly liquid, the solid part being comprised of the pure metal X. As the line DB is approached, more pure metal X is crystallized out until at the temperature represented by DB the remaining liquid solidifies as a eutectic.

As the percentage of metal Y in the alloy is increased, the pure metal X decreases in proportion to the eutectic, until at B the whole solid is eutectic. A still further increase in the constituent Y will form an alloy containing solid Y with eutectic. It follows that any point N in the area $DBGF$ represents a solid composed of pure metal X plus eutectic, any point on BG a solid alloy composed entirely of eutectic, and any point O in the area $BEHG$ a solid composed of pure metal Y plus eutectic.

Any point M in the area CBE will represent an alloy partly solid and partly liquid, the solid part being composed of the pure metal Y.

A constitutional diagram may very readily be obtained by freezing water and salt, and noting the arrests of temperature with an ordinary thermometer. Such a diagram is shown in Fig. 13.

An example of a series of alloys having a eutectic as an important constituent and approximating very closely to the ideal case given above is found in the lead-antimony alloy. Most alloys, however, depart more or less widely from the ideal case.
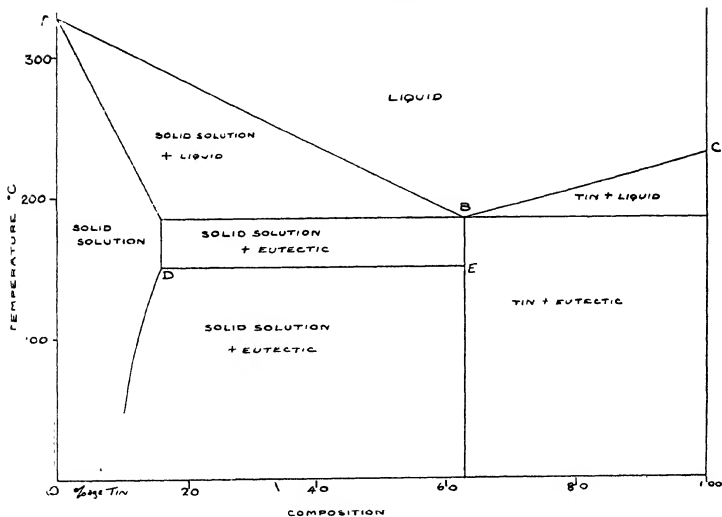
FIG. 14

The constitutional diagram for the lead-tin alloy is given in Fig. 14. On cooling from the liquid state, as the temperature falls below the line AB, a solid solution of lead containing a little tin begins to separate out, leaving the remaining liquid richer in tin, until the temperature of 183° C. is reached, when the remaining liquid forms a eutectic of 63 per cent. of tin and 37 per cent. of lead. When the mixture contains 63 per cent. of tin, the solid will be completely eutectic. For a greater percentage of tin, when the line BC is reached pure tin separates out until the remaining liquid contains 63 per cent. tin when the eutectic

is formed at the same temperature as before.   Another change, accompanied by thermal effects, takes place in the solid at 149° C. in alloys containing from 16 to 63 per cent. tin.   The explanation of this, put forward by Rosenhain, is that the critical temperature in question arises from a change in the metal lead which at that temperature alters its power of dissolving tin in the solid state.   From this it follows that the solid solution formed above the line *DE* will contain a different proportion of tin to that formed in the solid state below.

At the lead end of the series there is a considerable degree of solid solubility which takes place below the line *AD*, the lead and tin forming solid solutions, containing up to 16 per cent. tin.

Plumber's solder is a lead-tin alloy and contains about 67 per cent. of lead ; this it may be seen from the diagram is in the semi-liquid state for about 70° C., which enables the plumber to make the well-known wiped joints.

Steel.

The most important alloy used in the construction of aircraft is steel.

Steel is an alloy formed by the addition of carbon to iron.   Carbon is not a metal, but a metalloid.

The constitution of steel is complicated by the fact that iron can exist in two allotropic modifications ; that, is, there are changes in some of the properties of the iron without any change in the chemical composition.   A substance more generally known to have these modifications is sulphur, which can exist in several modifications, the best known of which are roll sulphur, flowers of sulphur and plastic sulphur.

The two modifications of iron are known as alpha iron (α iron) and gamma iron (γ iron).

α iron is the form of iron which exists when it has been cooled slowly to below 900° C. ; it is characterized by extreme ductility and has magnetic properties.   .

Between 900° C. and 760° C., iron used to be known as β iron, but is now generally recognized to be the same as α iron except for a loss in magnetic properties.

FIG. 15

FIG. 16

γ iron is formed above 900° C. It is non-magnetic, has a different crystal structure to and is harder than α iron. It is able to dissolve carbon up to 1·7 per cent., a property absent from α iron.

A cooling curve for pure iron showing the arrest where these changes take place is given in Fig. 16.

The constitutional diagram for the iron-carbon system is given in Fig. 15.

Above the line *ABC* the mixture is wholly liquid. At any point in the area *I* the mixture is composed of molten alloy and a solid solution of iron-carbide in γ iron known as austenite. In the area *II* the mixture is composed of molten alloy and graphite, graphite being an allotropic form of carbon which has a lack of cohesion with the grains of iron.

Below the line *ADB* the mixture is entirely solid. In the area *III* the alloy is wholly austenite, in the area *IV* austenite plus cementite, at *BJ* a eutectic of γ iron with cementite, and in the area *V* graphite plus austenite.

Cementite is a carbide of iron, corresponding to the chemical symbol Fe_3C, and consists of 6·6 per cent. carbon with 93·4 per cent. iron. It is very hard and brittle.   It frequently appears as a sharp needle-like structure showing white under the microscope.   It is seen clearly in 1·5 per cent. carbon steel, illustrated in Fig. 17 (see p. 12).

The portion of the diagram that has been already briefly explained does not concern this book, except in so far as it helps to make clear
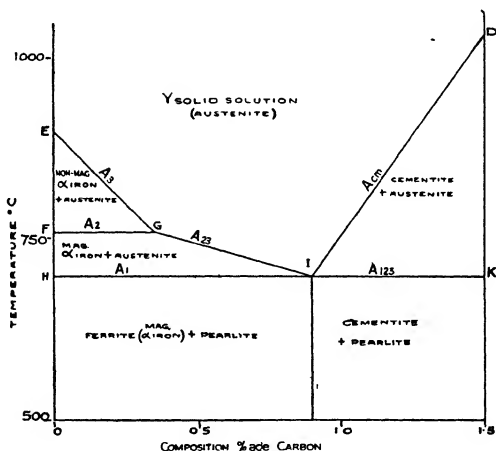

FIG. 18

the portion of the diagram in Fig. 18 which will now be explained in greater detail.

Above the line *EGID* the alloy is composed of a solid solution of cementite in γ iron, called austenite.

For steels containing up to 0·35 per cent. carbon, as the temperature drops below the line *EG* the γ iron begins to go out of solution and is transformed into non-magnetic α iron. Thus in the area *EGF* the steel is composed of non-magnetic α iron plus austenite, the proportion of austenite decreasing as the line *FG* is approached. At *FG* the non-magnetic α iron is transformed to the magnetic α iron, and the proportion of austenite, which is growing weaker in ferrite, is
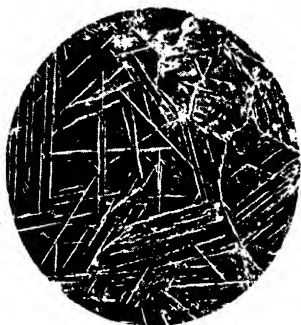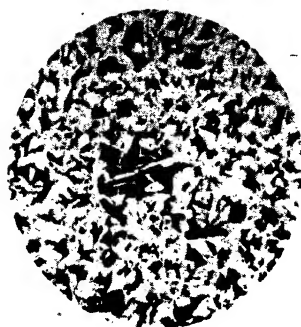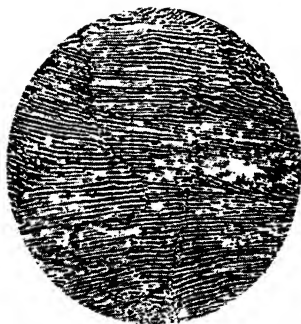
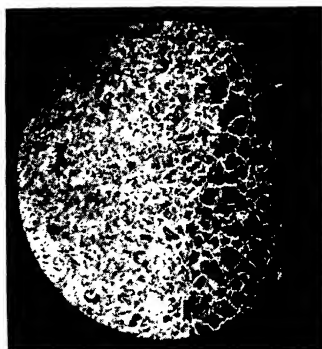Fig. 17

Fig. 19

Fig. 20

Fig. 28

still further reduced until on passing through the temperature *HI* the whole of the remaining austenite is transformed into eutectoid, called pearlite. A eutectoid is similar to a eutectic except that it forms out of a solid solution instead of a liquid.

The eutectoid, pearlite, contains 0·9 per cent. carbon and is a lamellar formation of alternate plates of cementite and α iron.

It is called pearlite because of its pearly lustre, due to the cementite and α iron showing light and dark respectively under the microscope.

Below the line *HI* the steel will be composed of pearlite plus magnetic α iron or ferrite. (Ferrite is the name given to iron free from carbon.) This is illustrated in Fig. 19 (see p. 12) for 0·3 per cent. carbon steel ; the ferrite showing white and the pearlite dark.

For steels containing 0·9 per cent. carbon the whole of the austenite will be transformed to pearlite at the point *I*. Such a steel is illustrated in Fig. 20 (see p. 12), and shows clearly the lamellar structure of pearlite.

A steel having a carbon content of from 0·35 to 0·9 per cent. will cool in a similar manner to one below 0·35 per cent. carbon except that there will be no change from γ iron to non-magnetic α iron, the γ iron changing direct to magnetic α iron on crossing the line *GI*.

Steels below 0·9 per cent. carbon content are called hypo-eutectoid steel and those with between 0·9 and 1·7 per cent. carbon hyper-eutectoid steels. Above this percentage are the cast irons.

Hyper-eutectoid steel will have its first change from pure austenite along the line *DI* where the austenite rejects cementite. This rejection continues until the line *IK* is reached, when the austenite, growing weaker in carbon, has reached the eutectoid composition and changed to pearlite—the final cooled steel containing cementite plus pearlite.

The microstructure of such a steel containing 1·5 per cent. carbon is shown in Fig. 17 ; the white needle-like structure is cementite and the dark ground mass pearlite.
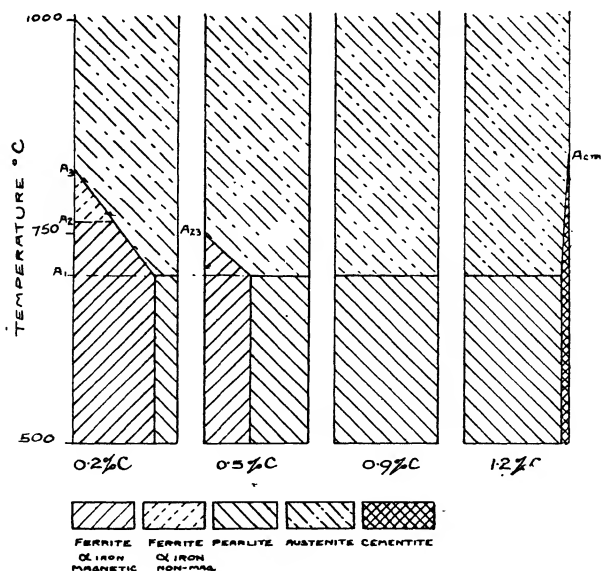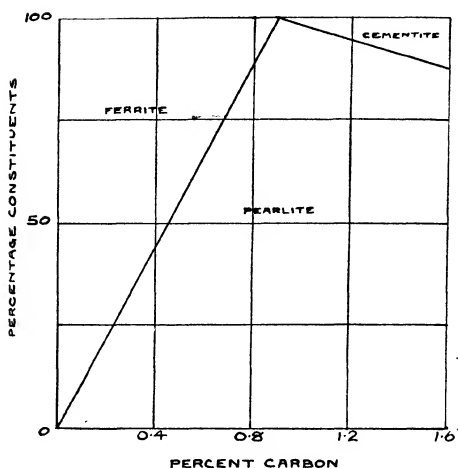

FIG. 21

FIG. 22.

Graphical illustrations of these changes are shown in Fig. 21, and in Fig. 22 a diagram showing the proportions of ferrite, pearlite and cementite in a slowly cooled steel containing up to 1.6 per cent. carbon.

Critical Points.

The temperatures at which the structural changes take place are known as the critical points and are denoted by the letter A followed by the figures 1, 2 or 3 to denote the point referred to and by the letters r or c, r signifying cooling and c heating.

So far the cooling curves have only been considered, but there will be corresponding changes on heating but at a slightly higher temperature, the difference in temperature depending partly on the rate of cooling and heating. This difference of temperature indicates that a certain time is required for the changes to take place, and if by very rapid cooling the time is not available it is quite conceivable that the change will not take place. As the critical points on cooling are not identical with those which occur during heating, they are distinguished by writing Ar for those which occur during cooling and Ac for those which occur during heating (r and c stand for the French words *refroidissement* and *chauffage*). The temperature at which the first change takes place on heating is called the lower critical point and written $Ac1$, and the corresponding point for cooling $Ar1$; this occurs when the line HI in the diagram is crossed. The change which takes place when the line FG is crossed is denoted by $Ac2$ or $Ar2$, and the upper critical point relating to the change which takes place when the line EG is crossed is written $Ac3$ or $Ar3$.

For steels having between 0·35 and 0·9 per cent. carbon content there will be only two changes : the upper, which occurs when the line GI is crossed, being denoted by $Ac23$ or $Ar23$. For steels having between 0·9 and 1·7 per cent. carbon the change corresponding to the line DI is designated Acm, and that corresponding to the line IK by $A1\ 2\ 3$, r and c being added to denote cooling or heating as before.

HEAT TREATMENT

So far we have only considered the changes which take place in slowly cooled steel. It will now be necessary to consider the effect of rapid cooling.

It has already been pointed out that a definite time is required for the changes to take place in the cooling steel. If there is only a very little or no carbon present, the γ iron changes to α iron simply by a rearrangement of the molecules in the crystals; there is no need for any movement of matter through relatively large distances. In changing from non-magnetic to magnetic α iron there is no change in the crystal structure, but merely a gain of magnetism.

When the steel contains an appreciable amount of carbon, the change involves a separation of ferrite from the austenite, necessitating a definite change of place of the carbide molecules in the solid alloy. This movement of matter through relatively large distances requires time, and if the time is not available the complete change of the austenite to ferrite and pearlite will not take place.

If a steel is heated above the critical point $Ac1$ and rapidly cooled by quenching in a suitable medium, such as water or oil, it will produce a steel of intermediate composition between ferrite plus austenite, and ferrite plus pearlite, depending on the carbon content and the rate of cooling.

These intermediate constituents have special characteristics which are of vital importance in the heat treatment of steel.

Martensite.—The first constituent in the decomposition of austenite is called martensite. It is characterized by a needle-like structure, which is very fine for a steel of about 0·9 per cent. quenched rapidly from just above $Ar1$, but gets coarser as the temperature from which it is quenched is increased.

A typical martensite structure is shown in Fig. 23 (see p. 16) for a nickel-chromium steel.

Of the constituents that may be obtained in carbon steels, martensite is the hardest, but also the most brittle. It has a very high tensile strength, but very little ductility. To produce it the steel must be quenched very rapidly from above the critical point. (Ductility is the ability to withstand a large amount of deformation without fracture. See Chapter VI.)

Troostite.—If the cooling is a little slower, the second constituent, called primary troostite, is formed; or if the martensite is reheated to a suitable temperature, decomposition will proceed and form what is known as secondary troostite. Troostite, whether primary or secondary, will be softer than martensite. Its principal properties will depend on how it is produced. Its appearance under the microscope will be dark and rounded as shown in Fig. 24 (see p. 16) for a 0·3 per cent. carbon steel.

Sorbite.—When the rate of cooling of austenite is still slower or the reheating temperature of martensite higher, another constituent, called sorbite, is produced. A sorbitic structure is illustrated in

Fig. 25, for the same steel as in Fig. 23.   This is usually considered to
be a variety of pearlite in which the ferrite and cementite are so finely
divided that they are not apparent under the microscope.   Sorbite,
which is softer than troostite, is much stronger than pearlite, though
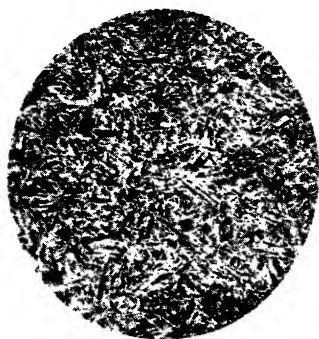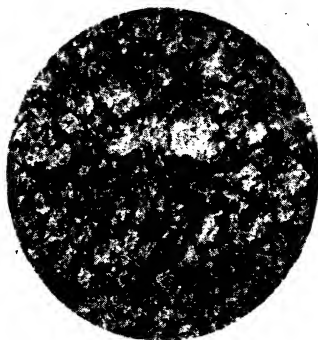slightly less ductile.

Fig. 23

Fig. 24
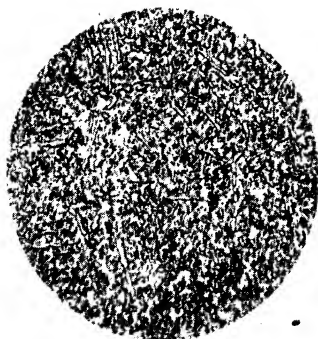
Fig. 25
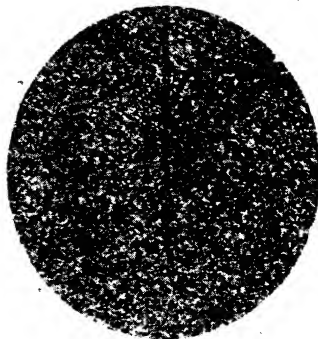
Fig. 26

Refining.

When steel is heated just above $Ac3$ it will be changed to austenite
of minimum crystal size and is said to be completely refined.   If,
however, the temperature is raised higher, or the steel is kept at a
temperature above $Ac3$ too long, the crystals will grow in size, leading
to a coarsening of the structure.   This coarsening reduces the toughness
and ductility of the steel.   As a general rule, small crystals stand for

Mechanical Treatment.

As the mechanical treatment of a metal is closely associated with its heat treatment, it will be necessary here to give a brief account of the effect produced on the structure by working the metal into the desired shape. Cold work, or mechanical treatment performed below the lower critical point, will not affect the crystal size, but it will distort the crystals. As a result the steel will become hard and brittle.

Hot work, which must be started well above the upper critical point, results in a refinement of the crystals. In working an ingot into a bar, it is passed through a number of pairs of rolls, each successively reducing it in size.

The pressure between each pair of rolls breaks up the crystals, which reform at the normal size corresponding to the temperature—*i.e.*, into smaller sizes—but if still above the upper critical point the crystals will grow in size after leaving the rolls, and if time is allowed may ultimately regain their former size.

In order, therefore, that the finished bar may have the desired small crystals, it is necessary to arrange that the bar leaves the last pair of rolls just before the critical point of the steel is reached, thus allowing no time for crystal growth.

Heat Treatment.

The object of heat treatment is to improve the physical properties so as to make the metal most suitable for the job required.

Annealing.—Annealing is a heat treatment which has for its object one or more of the following aims—

(1) To restore the perfectly crystalline structure, which may have been interfered with by cold working.

(2) To relieve the internal strains which may have been set up due to rapid cooling in previous heat treatment.

(3) To refine the grain and make the steel ductile.

(4) To make it soft for machining, etc.

For the full annealing of a hypo-eutectoid steel the temperature is slowly raised to just above the upper critical point, and slowly cooled. This, as has already been stated, gives a maximum refinement and at the same time fulfils the other three objects of annealing. A steel thus annealed has a structure of ferrite plus pearlite. If the steel is hyper-eutectoid, the final structure will be cementite plus pearlite, and the annealing temperature a little above $A123$, but below Acm.

This is because the Acm temperatures are so high that heating above them would produce grain growth and coarsen the structure.

By referring to Fig. 17, the reader may see the effect on the microstructure of annealing a 1·5 per cent. carbon steel. Fig. 17 shows the steel before the annealing process at 120 magnifications, and Fig. 26 (see p. 16) after annealing at 300 magnifications.

If the object is only to make the steel soft for machining, heating to just below the critical point and cooling in air will give the desired result.

If maximum ductility is required regardless of strength, the anneal is performed as follows—

Heat to just over $Ac3$, slow cool to just below $Ar1$, reheat to just over $Ac1$ and cool in the furnace. The first heating gives minimum grain size for the ferrite, and the second maximum resolution of the pearlite, both of which are preserved during cooling.

If strength and ductility are required, the steel is first heated to just over $Ac3$, air cooled to below $Ar1$, and reheated at a temperature just

below $Ac1$ and then slow cooled in air or the furnace. The air cooling after the first heating will, if the mass is not too large, produce sorbite, which will be retained during the second heating below the lower critical range.

This sorbitic steel will be stronger than in the previous case, where maximum ductility was required, but less ductile.

Normalizing.—The object of normalizing is to relieve the stresses and refine the crystal structure after it has been coarsened by previous hot working. This is carried out by heating to above the upper critical point and air cooling, except in the case of hyper-eutectoid steel, which is only heated to just above the lower critical point. The difference between normalized and annealed steel is that the former usually has a finer grain, and the pearlite is not lamellar as in annealed steel, but fine and almost structureless. This pearlite is called sorbitic pearlite and is stronger and harder than lamellar pearlite.

Hardening.—Hardening consists in heating the steel to some temperature above the lower critical point, and quenching in a suitable medium so as to retain the hard constituents, martensite and troostite. The temperature at which the steel is quenched, the quenching medium and the final result will depend upon the size of specimen and the carbon content. The less the carbon, the more difficult it is to retain
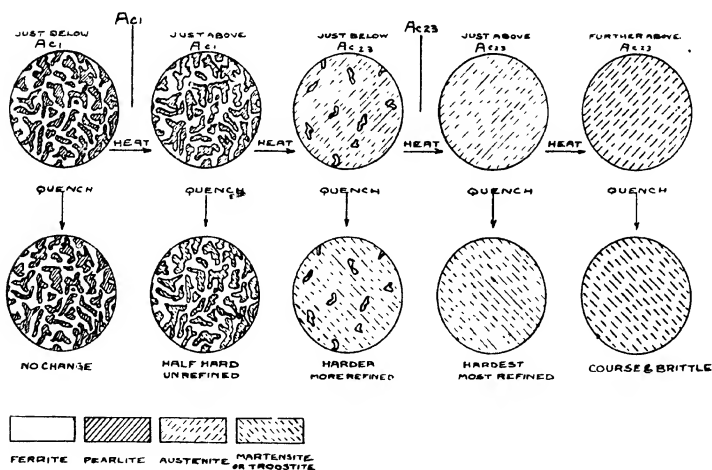

FIG. 27

the condition that obtains at the higher temperature ; it follows, therefore, that the smaller the percentage of carbon the more drastic must be the quenching.

Mild steels—*i.e.*, low carbon steels—respond so little to hardening that below a carbon content of, say, 0·4 per cent. it is not a practical proposition. The best steels for hardening are near the eutectoid composition of 0·9 per cent. carbon.

The usual quenching media in order of quickness are : liquid air, brine, water, oil and air. The first is not used commercially.

It will be as well to consider the hardening of a steel quenched at different temperatures. We will take a specimen of slowly cooled steel of about 0·6 per cent. carbon. This steel will consist of ferrite plus pearlite. Heating it to below the lower critical point will produce no change, neither will subsequent quenching.

Heating to just above $Ac1$ will change the pearlite to austenite, the free ferrite remaining unaffected. The steel will moreover retain its original grain size, the pearlite only being refined. Quenching will produce only a half-hard steel, the free ferrite remaining soft. Further heating below $Ac23$ will give a continued absorption of the ferrite by the austenite and an increased refinement. Quenching will give a harder and more refined steel, the hardness and refinement increasing as $Ac23$ is approached.

Heating to just above $Ac23$ will effect a complete absorption of ferrite and a minimum crystal size. Quenching at this temperature will give the hardest and most refined steel.

Any further heating above $Ac23$ will produce crystal growth, and the steel when quenched will be coarse and brittle. The above is illustrated graphically in Fig. 27. In the case of a hyper-eutectoid steel, heat to just above $A123$ and not Acm because of the rapid growth in grain size.

In the cases considered the quenching has been assumed to be very rapid. Slower quenching would mean less retention of the characteristics due to the heating.

For instance, a steel producing martensite when quenched in water from a certain temperature is likely to produce troostite when quenched from the same temperature in oil, and sorbite when quenched in the still slower medium air.

Tempering.—A hardened steel containing a martensitic structure will be brittle, and have internal strains due to the rapid cooling. Such a steel would be useless except for a few exceptional cases. It would certainly be of no use in a structure.

The object of tempering is to toughen the steel by relieving the strains and allowing the decomposition of the martensite to proceed so that troostite is formed, which is a typical constituent of tempered steels.

The tempering process is carried out by heating the hardened steel to a temperature below the lower critical point, and quenching it.

Tempering of a tool steel is usually carried out at any temperature from about 200° C. to 350° C., depending on the result required. The higher the temperature, the greater the transformation of martensite to troostite and the tougher the steel becomes. Should, however, the temperature be raised as high as 400° C., the troostite will begin to transform to sorbite, and the process is then often called toughening. The effect of toughening on the microstructure may be seen by referring to Figs. 23 and 25. Fig. 23 shows the hard martensitic steel and Fig. 25 the softer sorbitic steel obtained by toughening it.

In tempering it is more necessary to see that the specimen is heated to within very small limits of the required temperature as with hardening, but for a different reason—viz., small variations of temperature will give a marked difference to the physical properties obtained.

The best method to ensure a correct result is to immerse the specimen in a liquid bath which may be kept at a constant temperature all over.

Another method is to judge the degree of tempering by the colour of the specimen, which changes from yellow at 215° C. to blue tinged with green at 330° C., due to the formation of a surface film of oxide of iron.

Case Hardening.—The object of case hardening is to produce a steel with a hard surface to withstand abrasion and a tough core to withstand shock. A steel which is hardened throughout will be brittle

and unable to withstand shock, whilst one that is tough throughout will be too soft to withstand wear.

It will be remembered that a low carbon steel will not respond to heat treatment as well as one near the eutectoid composition. If, therefore, a piece of steel could be made to vary in composition from high carbon at the surface to low carbon at the centre, heat treatment would produce a specimen hard and brittle on the surface and tough at the centre. This is what happens in case hardening, but there must be the previous operation of producing a surface of high carbon content. This is known as case carburizing, and is carried out in the following manner : A low carbon steel is heated above the upper critical temperature while in contact with a mixture of carbonaceous materials. This carburizing mixture (usually carbonates plus free carbon) forms carbon monoxide, which penetrates the case of the steel. This, being in the austenitic state, will dissolve the carbon from the carbon monoxide—*i.e.*, carbon monoxide plus γ iron forms cementite plus carbon dioxide. The depth to which the carbon penetrates depends on the time of heating, and is not as a rule over $\frac{1}{16}$ inch.

The carburized steel will not have the necessary hardness due to its slow cooling ; the case will also be coarse-grained, due to the length of time it has been above the upper critical point.

To harden the surface and refine the whole specimen, the steel is first heated to above the upper critical point of the core and quenched, then heated to just above the lower critical point and quenched. The first operation will refine the core, and the second harden and refine the case without affecting the core.

Fig. 28 (see p. 12) shows a piece of case-hardened nickel steel. The dark side of the photo is the hardened case and the light side the core. The penetration of the carbon from the outside may be clearly seen.

Nitriding

Nitriding is a fairly new process whereby steels are given an extra-ordinarily hard surface at comparatively low temperatures, without quenching.

Special steels, containing chromium and aluminium or chromium and molybdenum, which act as inhibitors so that the depth of hardness can be controlled, are used, and may be heat treated in the usual way to get the desired core properties. Components made from this steel are placed in a sealed box, through which ammonia gas is circulated, and the whole heated to 500° C. in a furnace.

The depth of hardness depends upon the duration of treatment, 95 hours giving a depth of about 0·03 inch. If desired, part only of the surface may be hardened, the parts to be left soft being first tinned.

Austenitic valve steel may be nitrided after being first etched with a solution of sulphuric acid, and copper plated to protect the surface.

After hardening about two-thousandths of an inch are removed by buffing, to compensate for a slight increase in dimensions, and to remove the most brittle outer film.

The advantages of this process are—

(1) The very hard surface obtained

(2) The low temperature at which the process is carried out preventing distortion which is an important consideration in cases of such large thin parts as sleeve valves

(3) The temperature being well below the lower critical point previous heat treatment is not interfered with

(4) The case has a high resistance to corrosion.

CHAPTER IV

ALLOY STEELS AND LIGHT ALLOYS

Impurities in Steel.

The two essential elements of steel are iron and carbon. There are always other elements present, but these are impurities which are not essential to the formation of the steel. The chief impurities are sulphur, manganese, silicon and phosphorus.

The sulphur comes chiefly from the coke used in the furnace, and should not exceed 0·06 per cent. It is detrimental, imparting brittleness to the steel when hot worked. Manganese is useful in steel, due to its neutralizing effect on sulphur. Sulphur combines with manganese in preference to iron, so that if manganese is present the sulphur forms manganese sulphide instead of iron sulphide. Although the manganese sulphide is undesirable, it does not weaken or embrittle the steel so much as iron sulphide. It is usual to have about six times as much manganese as sulphur present, in order to be sure that no iron sulphide is formed.

Silicon is present partly in the slag and partly in the solid solution, but in some cases partly combined with the iron to form silicide of iron. There is usually less than 0·3 per cent. and as such is not injurious, but improves the soundness by diminishing blowholes, and in larger proportions modifies the properties of the steel by entering into solid solution.

Phosphorus also enters into the slag, and if in the steel in sufficient quantity might combine with the iron to form phosphide of iron. It should be kept as low as possible, for it is liable to render the metal brittle and lowers its resistance to shock.

Alloy Steels.

Alloy steels are those containing one or more elements other than iron and carbon, sufficient to modify its physical properties. A big advantage of all alloy steels is that less rapid quenching is necessary, owing to the slower critical changes, thus permitting slow and more uniform cooling, with greater penetration of hardening effect and less risk of cracking.

Another advantage is increase of strength, in some cases to as high as 150 ton/sq. in., after suitable heat treatment. For structural purposes the maximum strength of pure carbon steel is about 45 ton/sq. in. Greater strength may be obtained by using lower tempering temperatures, but the steel is then too brittle. High carbon steels may be made extremely hard for use as cutting tools, but the hardness is destroyed if the tool becomes overheated. Alloy tool steel will retain its hardness even when red hot, so the machining processes may be carried out at higher speeds. Magnets made from alloy steel are more powerful and retain their magnetism better than those made from carbon steel. Corrosion resistance may be very much increased, principally due to the presence of chromium.

Nickel Steels.—The most numerous of the alloy steels are those containing nickel as an alloying element. Nickel increases the strength without appreciably affecting the ductility of the steel ; it lowers the

critical points and also reduces the carbon content of the eutectoid. A 3·5 per cent. nickel steel will form pure eutectoid with about 0·75 per cent. carbon, instead of 0·9 per cent. as with a carbon steel. The nickel reduces the crystal size, which is the case with all alloy steels ; the alloy providing extra points from which the crystals may grow, produces more and therefore smaller crystals.

When the nickel is over 25 per cent., the lower critical point will be below room temperature ; it follows, therefore, that without heat treatment the steel will have an austenitic structure—i.e., the iron is γ iron and non-magnetic. It is used for parts through which a magnetic field should be prevented from passing, such as the rotor of a polar inductor magneto. Between 10 and 25 per cent. the steel will have a martensitic structure, and below 10 per cent. a pearlitic structure. A steel of low carbon content and having 36 per cent. nickel is known as " invar metal " ; it has a very low coefficient of expansion, and as such is much used for clock pendulums, measuring instruments, etc.

The martensitic nickel steel has no practical value, as it would be impossible to machine it.

The principal pearlitic nickel steels are those containing from 3 to 4 per cent. nickel and from 0·2 to 0·5 per cent. carbon. These steels are hardened and then tempered or toughened according to instructions provided by their makers, which vary somewhat for each steel. In principle the treatment is much the same as for carbon steel, except that for hardening the steel it is usual to employ oil quenching, due to the nickel retarding the change from austenite to martensite or pearlite.

Pearlitic nickel steel is much used for case-hardening, a hard case being produced after carburizing with comparatively slow cooling, and the presence of nickel preventing large grain growth during carburization.

Chromium Steels.—Chromium steels may be divided up into three classes, depending on the chromium content. Those with under 2 per cent. chromium are used where strength and hardness are required, those having from 2 to 4 per cent. chromium may be used for permanent magnets, and those having from 10 to 20 per cent. chromium are the stainless steels, so called because of their resistance to corrosion.

Chromium forms a carbide, which imparts to it great hardness and strength ; it retards the change of the austenite to pearlite and so gives a greater depth of hardness in a thick specimen. Chromium produces a fine grain size, giving added strength with no appreciable loss of ductility. It raises the critical points and reduces the carbon content of the eutectoid.

Stainless steel was first used for cutlery, but when the war broke out in 1914 its use was extended to aero-engine valves. It contained 0·35 per cent. carbon and from 12 to 14 per cent. chromium. For some years this type was the only one available for all purposes. In recent years, however, improvements have been made by altering the proportions of chromium and carbon, and the addition of another alloying element—nickel.

" Staybrite," which is the commercial name for the stainless steel containing nickel, is an austenitic steel, and has for its approximate composition—

Carbon	...	...	...	...	0·15 per cent.
Manganese	...	...	...	...	0·3 „
Silicon	...	...	...	...	0·7 „
Chromium	...	...	...	...	18·0 „
Nickel	...	...	...	...	8·0 „

It is better than the original stainless steel from the point of view of corrosion resistance. The original steel has to be hardened in order to exhibit its maximum resistance to corrosion, whereas

"Staybrite," generally speaking, is not affected in its resistance to corrosion by heat treatment. "Staybrite" also has a higher ductility. The strength of this steel is low, but may be considerably increased by cold drawing and rolling operations.

A stainless steel of greater strength but less ductile and not so satisfactory as regards welding has an approximate composition of—

Carbon	...	...	...	...	0·2 per cent.
Manganese ...	...	...	...	0·15	,,
Silicon	...	...	...	...	0·15	,,
Chromium ...	...	...	...	18·0	,,
Nickel	...	...	...	...	2·0	,,

Stainless iron, which is really a martensitic steel, has a greater ductility than the higher carbon material, but does not possess the superior corrosion-resistance of the "Staybrite" steel. Its composition varies greatly, an average being about—

Carbon	...	...	...	...	0·1 per cent.
Manganese ...	...	...	...	0·3	,,
Silicon	...	...	...	...	0·2	,,
Chromium ...	...	...	...	13·0	,,

Nickel-Chromium Steels.—Of all the alloy steels, those containing chromium up to 1·5 per cent. and nickel up to 3·75 per cent. are the most generally used. The best results are obtained when the ratio of nickel to chromium is about 3.

The effect of the two alloying elements is to give strength and hardness from the chromium together with toughness and ductility from the nickel.

The usual heat treatment for these steels is to heat to about 820° C., soak for about three-quarters of an hour, and quench in oil ; then temper or toughen by reheating to a temperature depending on the steel and the physical properties required. It is usual for the manufacturers to provide suitable charts showing how the physical properties vary with the reheating temperature. Such a chart is reproduced in Fig. 28A. It will be noticed that the strength and hardness fall off as the reheating temperature is increased, but that the ductility and resistance to shock increases. If strength and hardness are wanted and ductility is a secondary consideration, the best reheating temperature would be about 200° C. ; if, on the other hand, ductility is the primary consideration, then 650° C. would be the best temperature. It is usual, however, to want both strength and ductility, and therefore a compromise must be made, in which case the most practical reheating temperature would be about 500° C.

Steels containing over 3 per cent. nickel and 1 per cent. chromium, and in which the nickel + chromium + carbon is over 5 per cent., have the important property of air hardening—*i.e.*, they do not require quenching—and are therefore less likely to distort and show greater depth of hardening.

Other Alloying Elements.

Manganese is added to steels to clean and toughen them by eliminating ferrous oxide and sulphur, which are harmful impurities.

Small amounts of silicon may be added to produce a sound metal and so increase its fatigue resistance. Large amounts are not used, as they would increase the grain size.

Molybdenum is used to toughen the steel by refining the grain and to increase the fatigue strength. It also gives to a steel the property of air hardening.

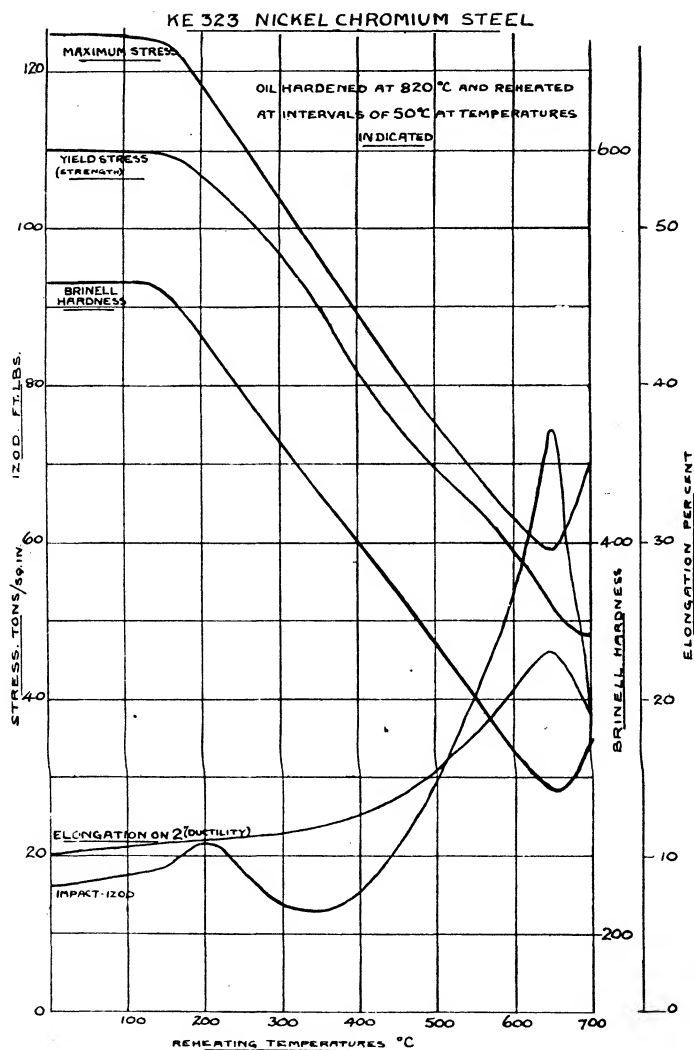Vanadium has similar effects to molybdenum. It may be said to

FIG. 28A

intensify the effect of the chromium. It is very costly and so percentages over 0·2 per cent. are seldom used.

Light Alloys.

Aluminium.—Commercially pure aluminium is a comparatively weak metal ; it is, however, very light, its specific gravity being only about one-third of that of steel. Due to its high ductility, it is a very suitable metal for non-structural parts, such as engine cowlings, petrol tanks, etc.

Duralumin.—With the rapid advance of aircraft and the consequent demand for materials having a high strength compared with their weight, metallurgists turned to aluminium with the intention of alloying it with a small quantity of other metals, so as to increase its strength without appreciably increasing its weight. The result has been the production of several light alloys, of which the most important and widely used is duralumin.

Duralumin is an aluminium alloy having the following approximate composition—

Copper	...	...	...	...	4·0 per cent.
Magnesium		...	...	...	0·6 ,,
Manganese	...	...	...	...	0·6 ,,
Iron	...	...	...	...	0·3 ,,
Silicon	...	...	...	...	0·2 ,,
Aluminium	...	...	...	...	The rest.

Duralumin may be normalized, by which its strength will be increased from 18 tons per square inch to 26 tons per square inch. It is heated in a salt bath at a temperature of 480° C. and then quenched in water.

It is not safe to heat it in a furnace, as the temperature is never sufficiently uniform, so that it is liable to exceed 520° C. in places, with the result that if the specimen is sheet it will quite likely fall to pieces on being taken from the furnace, and in any case will blister and crack on being quenched. In no case should the temperature be allowed to exceed 490° C.

After heat treatment the metal is still soft, but it will get harder rapidly at first and then more slowly until at the end of a week no appreciable hardening can be noticed, and the metal has reached its hardest and strongest state, at which it will remain. This peculiar slow hardening after normalizing is known as " ageing." It is considered due to the presence of a constituent more soluble at high temperatures than at low. On heating, this constituent is dissolved by the solid alloy, but when quenched there is not time for it to be precipitated ; the cold duralumin therefore contains a solid solution supersaturated with this constituent, which is slowly precipitated during ageing. The precipitate disturbs the crystal form, causing hardening of the material.

Duralumin may be softened by annealing, which consists of heating to 350° C. for about fifteen minutes and quenching in water. It then remains soft and can be worked for a considerable time afterwards ; but if severe work is to be done on it, it is best that it should be carried out within twenty-four hours of annealing. When formed into the required shape, the member should be normalized as explained above.

It is not, however, necessary to anneal duralumin when it can be worked quickly ; it can be normalized and then worked cold within two hours—*i.e.*, while still soft.

When it is important to avoid distortion quenching may be carried out in oil. In this case heating must be carried out in a furnace and *not* in a salt bath, as the use of the latter in conjunction with oil quenching is very dangerous.

The fully age-hardened alloy should not be heated above 65° C., because this affects its properties. For this reason protective processes such as stove enamelling, involving high temperatures, should not be used.

" Y " Alloy.—This is another aluminium alloy, used for castings, which is similar to duralumin with the addition of nickel and more magnesium. Its approximate composition is—

Copper	...	...	...	...	4·0 per cent.
Nickel	...	...	...	...	2·0 ,,
Magnesium		...	...	...	1·5 ,,
Silicon	...	...	...	...	0·3 ,,
Iron	...	...	...	...	0·3 ,,
Aluminium		...	...	...	The rest.

It may be normalized by quenching in boiling water from a temperature of 520° C. After ageing for five days, it will have a maximum strength of about 22 tons per square inch—*i.e.*, not quite as high as duralumin. It has the advantage of a cleaner structure than duralumin and is not so liable to corrode. It may be softened and worked in the same way as duralumin.

Magnesium Alloy.—Although magnesium is a new metal for structural purposes, it is likely to play an important part in the development of light alloys on account of its low specific gravity of only 1·7.

The object of alloying small proportions of other metals with magnesium is, as for aluminium, to increase its strength without appreciably increasing its weight.

The most widely known magnesium alloy is called Elektron. It contains zinc as the main alloying element and small quantities of aluminium, manganese, iron, etc. This alloy has nearly the strength of duralumin, but is only two-thirds the weight, which is its great advantage. It is not usually heat treated as its reaction is slight.

The main obstacle to its use in the construction of aircraft is its liability to corrode, especially under the influence of sea-water. Its inflammability has also probably influenced some designers against it. It is, however, unlikely to become ignited except by intense heat, when used in the thickness that would be required for a structural member. Precautions should be taken to guard against fire when machining this metal as thin turnings and shavings are easily ignited. They should not be allowed to accumulate, owing to the risk of spontaneous ignition should they become damp.

Specified Materials.

The materials used in aircraft are far too numerous to consider individually in this book. The student who wishes to know the composition, heat treatment or physical properties of any specified material is referred to the " Handbook of Aeronautics " or the British Standard and D.T.D. Specifications for Aircraft Materials.

The factors which influence selection are—

(1) Strength in relation to weight.
(2) Resistance to wear and corrosion.
(3) Cost of (*a*) material, (*b*) heat treatment and equipment for working.
(4) Supply in peace and war.

Corrosion.

Corrosion is a chemical action which takes place when moisture, and more particularly salt water, is present, resulting in loss of metal either on the surface or along the crystal boundaries inside the metal. It is

accelerated, due to an electrolytic action, when dissimilar metals are in contact. For this reason dissimilar metals are always separated by a suitable insulating material.

Corrosion is a very vital consideration in all structures, but especially with seaplanes which come into contact with the highly corrosive power of sea-water. The effect of corrosion is to reduce the static strength of a member, but more important still is the fact that a very minute amount of corrosion will considerably reduce a material's resistance to fatigue.*

In general engineering structures a margin of excess material is provided to make allowance for whatever corrosion is likely to occur, and this considerably increases the structure weight. In the case of aircraft structures it is highly undesirable that this extra weight should be added ; at the same time it is necessary that metal parts should be sufficiently resistant to corrosion to ensure that no appreciable corrosion takes place during the normal life of the aircraft.

The aircraft constructor endeavours to prevent corrosion by the use of protective coatings and stainless steels, rather than to allow for it by an extra margin of material. Of course, the protective coating will add to the weight, and the problem is to obtain one which is both effective and light.

Protection of Steel.—Stainless steels are now much used in aircraft. Due to their very high resistance to corrosion, no protective coating need be applied. This high degree of corrosion resistance is attained by the protective effect of a film, which forms on the exposed surface, due to the chromium in the steel.

For other steels stove enamelling has been found to give satisfactory results, and the enamel adheres well to the part. Cellulose air-drying enamels have been used, but it is difficult to obtain a sufficient degree of adhesion in this case.

The main disadvantage of enamels is that they are heavy, and may weigh as much as $1\frac{1}{2}$ oz. per square yard of surface.

Cadmium plating is now extensively used to protect aircraft parts. This is an electrolytic process, in which the thoroughly cleaned parts are immersed in a bath of cadmium and potassium cyanide and water. The parts to be plated act as the cathodes, and the anodes are sheets of cadmium.

Zinc plating by a similar process is also very much used. In this case the bath is made up of zinc and sodium cyanide, caustic soda and water, and the anodes are of zinc. The zinc deposits are harder than those of cadmium. more durable, but give a more brittle surface. The process is cheaper than cadmium.

These processes usually cause a slight degree of embrittlement of the steel, which is removed by heating for thirty minutes at a temperature of between 100° C. and 200° C.

Protection of Aluminium Alloys.—Aluminium alloys are very liable to corrode when exposed to sea-water or a salt-laden atmosphere. Contact with other metals should be avoided, as this considerably accelerates corrosion, due to electro-chemical action. This is the case when any two dissimilar metals are in contact.

The main danger in the use of aluminium alloys is that the corrosion is often not very apparent on the surface, but takes place along the boundaries of the crystals inside the metal.

Protective measures include the application of enamels and varnishes, and anodic oxidation.

The best results with enamels have been obtained when an aluminium-cellulose enamel is applied over an undercoat of oil varnish.

*See " Fatigue of Metals," Chapter VI.

A still better result is obtained by an electrolytic process, termed anodic oxidation, whereby an oxide film is produced on the surface of the metal. The protective effect of the treatment is very much increased by greasing the surface with lanolin after treatment. If, however, the part is inaccessible, and so cannot conveniently be periodically re-greased, by far the best result is obtained by the application of a coat of enamel over the oxide film.

Pure aluminium has a high resistance to corrosion. This has led to the use of Alclad, which is a duralumin sheet coated with a thin layer of aluminium. The duralumin provides the strength and the aluminium the protection, which even extends to the cut edges.

Protection of Magnesium Alloys.—Magnesium alloys possess a very low resistance to corrosion, and at present there is no method of protection sufficient to cope with the severe conditions prevailing in those parts of marine aircraft which come into contact with the sea-water. For less severely exposed parts, sufficient protection is obtained by immersing the part in a chromate bath, after first cleaning it with 10 per cent. nitric acid. This process produces a thin film on the surface, which improves the corrosion resistance and forms a suitable base for the application of enamels. Good results are obtained when this chromate treatment is supplemented by the application of oil or cellulose enamel.

CHAPTER V

STRAIN, STRESS AND ELASTICITY

Strain.

All bodies are altered in shape by the forces acting on them. This distortion is called strain. In the author's dictionary strain is defined as " exert to the utmost ; injure." This, however, is not the technical meaning of strain. The smallest force will distort a body and the body will· be strained. Whether the body will return to its original shape, or not, on the force being removed does not affect the issue. In fact, we may say that all bodies are strained, if only by the force of their own weight.

As a rule the strain due to the weight of the body is negligible, but, as will be seen later, in some cases it is important.

Stress.*

The internal actions and reactions in a body that are called into play to resist strain (and transmit force) are called stress.

Having read Chapter I, it will be fairly obvious that these internal actions and reactions are due to the cohesive forces between the molecules. It is hoped that the following explanation, though incomplete, will give the student a picture by which he may understand the phenomena of stress and strain.

Imagine a wire has a pull in it of, say, P. At any cross-section of the wire this force must be divided between the molecules on one side of the section, and they must transfer the force to the molecules on the other side. Assuming there are x molecules on each side of the section and that they are opposite each other, then each molecule on one side will have a force of P/x pulling one way, and those on the other side a force of P/x pulling the opposite way. Remembering that the molecules are kept a definite˙distance apart due to their vibration, but are held at that distance in a solid due to their cohesion, any extra force tending to pull them apart will mean that they are no longer in equilibrium. They will therefore travel further apart until their vibration effect plus the force applied is equal and opposite to the cohesive force. This will be possible, due to the effect of vibration decreasing with the extra distance. If the force P is decreased, the molecules will be pulled nearer by cohesion, but if increased they will be pulled further apart until the cohesion is no longer sufficient to overcome the applied force, and the wire will break.

These actions and reactions will, of course, take place throughout the length of the wire, and so will the increased distances apart of the molecules ; it should therefore be obvious that the wire will be increased in length—*i.e.*, strained. Although the increase in length is not very apparent in a small length of wire, it can readily be seen in a piece of elastic.

The actions and reactions, or stress, will, of course, increase with the pull and decrease with the cross-sectional area ; thus in the case we have taken, if the wire had double the area, then there would be $2x$ molecules at the cross-section considered, and the reaction on each molecule due to the force P would be $P/2x$, or half of what it was before.

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* It is not considered necessary to confuse the student with the difference between Unit and Total Stress, as nobody thinks of stress otherwise than as unit stress in practice.

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' The foregoing should have also made it clear that for a given stress there will be a definite amount of strain. In the case considered the stress is the same throughout the wire, but, as will be seen later, this is not so in all cases.

Elasticity.

If the loading is such that the molecules have only been moved further apart, or nearer together, when the force which has caused this movement is removed, the cohesive forces will bring them back to their original positions, and in consequence the body will no longer be strained. If, however, there has been a displacement of the molecules, which might occur due to some giving way while others held, the body will not return to its original shape when the load is removed. This is illustrated diagrammatically in Fig. 29.
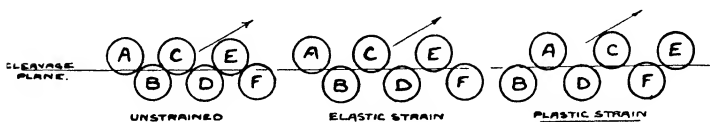
FIG. 29

The material of a body is said to be elastic if the whole of the strain disappears when the force which has produced it is removed. All materials that are used for structures are elastic to within certain limits of stress ; if stressed beyond this limit and the stress removed, they will only return partly to their original shape. The strain that remains is called " permanent set."

A material which retains the whole of the strain after the force producing it is removed is said to be perfectly plastic. A plastic body under stress acts much in the same way as a liquid, and may be said to flow into another shape.

Hooke's Law states that for an elastic material the strain is proportional to the stress producing it.

Modulus of Elasticity.

If Hooke's Law is true, we may write for a given elastic material and type of stress—

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant.}$$

This constant is known as the modulus or coefficient of elasticity of the material. This modulus will be a measurement of the tendency of a material to resist strain. Thus, if two similar bodies under the same stress are made of different materials, such that the modulus of elasticity of one is twice that of the other, then the body made of the material with the higher modulus will only distort half as much as the other.

Measurement of Stress.

We have already seen that the stress is proportional to the force causing stress, and inversely proportional to the area transmitting the force. The stress is measured as—

$$\frac{\text{Force}}{\text{Area transmitting force}}$$

If the stress is not uniformly distributed, the intensity of stress at any point will equal—

$$\frac{\text{Force on indefinitely small area at point}}{\text{Indefinitely small area}}$$

Measurement of Strain.

Strain is measured as the unit distortion of a body. Thus if a body 3 inches long increased in length 0·1 inch, the strain would be measured as the increase in length of 1 inch only—*i.e.*, $\dfrac{0·1}{3}$

Kinds of Stress and Strain.

There are three principal kinds of stress : tensile, compressive and shear—with three corresponding strains : extension, contraction and shear or slide. A body may be under the influence of one or more of these stresses.

Tension.—A body is said to be in a state of tensile stress when the force exerted tends to pull the molecules apart. A simple example is the case of the tie rod in Fig. 30.

$$\text{Tensile stress} = \frac{\text{Pull}}{\text{Area transmitting pull}}$$

If f = stress,

P = force,

A = area resisting force,

we may write $f = \dfrac{P}{A}$ lb./sq. inch or tons/sq. inch.
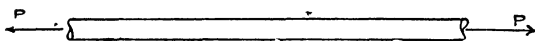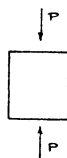
Fig. 30   Fig. 31

In dealing with pure tensile stress, A is usually the area of cross-section of the member.

The tensile strain is measured as the extension per unit length—

i.e., tensile strain $= \dfrac{\text{increase in length}}{\text{original length}} = \dfrac{e}{L}$

where e = increase in length,

L = original length.

The modulus of elasticity for pure tension is called Young's Modulus and is always denoted by the letter E.

$$E = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\dfrac{P}{A}}{\dfrac{e}{L}} = \frac{PL}{Ae} \text{ lb./sq. inch or tons/sq. inch.}$$

Compression.—A body is said to be in a state of compressive stress

when the force exerted tends to force the molecules nearer together. A simple example is given in the case of the block, Fig. 31.

$$\text{Compressive stress} = \frac{\text{Compressive force}}{\text{Area transmitting compression}} \text{ or}$$

$$f = \frac{P}{A} \text{ lb./sq. inch or tons/sq. inch.}$$

Compressive strain is measured as the contraction per unit length—

$$\textit{i.e.,} \text{ Compressive strain} = \frac{\text{decrease of length}}{\text{original length}}$$

$$= \frac{e}{L}$$

where e = decrease in length.

The modulus of elasticity for pure compression has the same value as for tension, except for a very few materials which need not be considered here.

Pure compression only occurs when a body is short compared to its least transverse dimension. Long struts are dealt with in Chapter XIII.

POISSON'S RATIO.—When a body is strained in one direction, there will also be an opposite kind of strain in every perpendicular direction. Thus if a tie rod is extended longitudinally, there will also be a contraction laterally, as illustrated in Fig. 32 (*a*). This may be explained by imagining the molecules before strain arranged as in Fig. 32 (*b*). They are kept at these mean distances apart due to their cohesion and vibration ; if now they are strained longitudinally only, it will be seen from Fig. 32 (*d*) that they are further apart in all directions. As, however, there is only a longitudinal force pulling them further apart, and no force to keep them further apart laterally, cohesion will at once make them contract in the lateral direction.
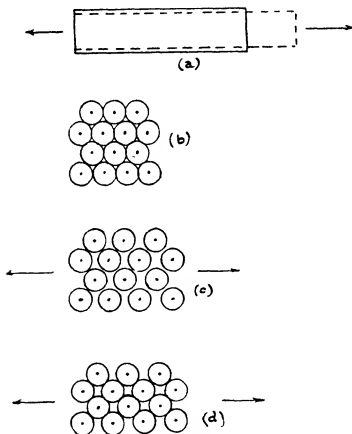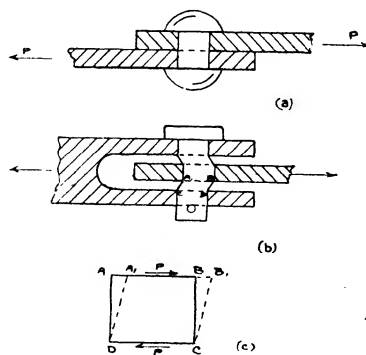
FIG. 32   FIG. 33

This lateral strain is proportional to the longitudinal strain.

The ratio $\dfrac{\text{lateral strain}}{\text{longitudinal strain}}$ is a constant for any given material, and is known as Poisson's Ratio. It is generally denoted by the fraction $\dfrac{1}{m}$. m varies from 3 to 4 for most materials.

Shear.—A body is said to be in a state of shear stress when the molecules tend to move over each other, such that two sections of the body in contact tend to move in opposite directions parallel to the surface of contact.

A simple example is that of the rivets, Fig. 33 (a and b), and the block, Fig. 33 (c).

$$\text{Shear stress} = \frac{\text{Shear force}}{\text{Area transmitting shear force}}$$

$$= \frac{P}{A} \text{ lb./sq. inch or tons/sq. inch.}$$

A member, such as a rivet, is said to be in single shear when the shear force is transmitted by one cross-section of the rivet as in Fig. 33 (a), and in double shear when transmitted by two cross-sections as in Fig. 33 (b).

In case (a) the amount of rivet under shear strain is too small to consider clearly ; it exists merely on the portion of the rivet between the two plates, which might in this case be said to be infinitely small. In cases (b) and (c) we can more readily study the effect of strain, it being on a greater length—*i.e.*, on the portion *ABCD*.

Shear strain is measured as the angular displacement (in radians) produced by shear force.

Consider the block *ABCD*—Fig. 33 (c)—fixed along *DC*. Under a shear force *P* the block will be distorted such that *AB* takes up the position A_1B_1. The angular displacement α radians is the shear strain. It is independent of the height *AD*.

As α is small, we may write

$$\text{Shear strain} = \alpha \text{ radians} = \frac{BB^1}{BC}$$

It will be noticed that the shear strain, and therefore the shear stress, is not affected by the length *AD*. If, however, the length *AD* becomes appreciable, the rivet in case (b) will have to withstand stresses due to bending as well as shear.

Modulus of Rigidity.

This is the shear modulus of elasticity, and is equal to

$$\frac{\text{Shear stress}}{\text{Shear strain}} = \frac{P}{A\alpha} = C.$$

Modulus of Rigidity is usually denoted by the letter *C*.

Complementary Shear Stress.

A shear force cannot exist on a body in one direction without a balancing shear force in a direction at right-angles.

Thus if the body *ABCD* (Fig. 34a) is under shear stress due to the forces *P* along *AB* and *CD*, there will be an unbalanced moment
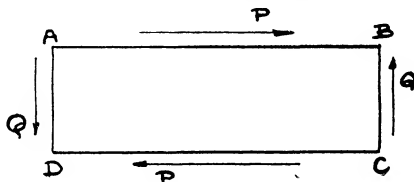
Fig. 34 (a)

$P \cdot AD$ tending to turn the body. In order that it may be in equilibrium there must be balancing forces Q at right-angles to P along AD and CB exerting an equal and opposite couple $Q \cdot DC$.

Let the shear stress due to $P = f$,

and that due to $Q = q$.

If the width of the block equals w,

$$f = \frac{P}{\text{Area transmitting shear}}$$

$$f = P/AB \cdot w \text{ or } P = f \cdot AB \cdot w$$

and $q = Q/AD \cdot w$ or $Q = q \cdot AD \cdot w$.

Taking moments about D,

$$Q \cdot AB = P \cdot AD.$$

Substituting for Q and P

$$q \cdot AD \cdot w \cdot AB = f \cdot AB \cdot w \cdot AD.$$

$$\therefore q = f.$$

That is, the shear stresses which must exist in a body at right angles to each other are equal.

Normal Stresses.

Consider a square portion $ABCD$, taken from any body subjected to shear, such that the shear forces S act along the faces, as shown in Fig. 34(b).
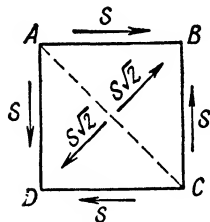
Let the areas of the faces $= a$, then the shear stress $= S/a$.

Now consider the forces on the diagonal AC, and we see that the shear forces combine to subject AC to a tensile force $S\sqrt{2}$. As the area at AC is $a\sqrt{2}$, the tensile stress across AC is—

$$\frac{S\sqrt{2}}{a\sqrt{2}} = \frac{S}{a} = \text{Shear stress.}$$

Fig. 34 (b)

In the same way it may be shown that there is an equal compressive stress across the diagonal BD. Thus whenever a body is subjected to a shear stress there is also present equal tensile and compressive stresses, at 45° to the shear stress.

Bulk Modulus.

The bulk modulus is that which expresses the relation between stress and change in unit volume when a body is subjected to equal stresses on all faces, such as when a body is under pressure. It is denoted by the letter K.

$$K = \frac{\text{Stress}}{\text{Volumetric strain}}$$

$$\text{Stress} = \frac{\text{Normal load}}{\text{Area transmitting load}}$$

and in the case of a pressure will equal the pressure.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

Oblique Stresses.

If the stress in a body is required in any cross-section other than that at right-angles to the applied load, the load may be resolved into rectangular components at right-angles and parallel to the section considered.

Let the rectangular bar of uniform section (Fig. 35) be subjected to a pull P. The stress f on any cross-section AB at right-angles to P will equal $\dfrac{P}{A}$, where A is the cross-sectional area at AB.

To find the stress across the section DC, resolve P at right-angles and parallel to DC.

In the triangle of forces EFG, if EF represents P, then EG represents the component of P at right-angles to DC. This will be a tensile load. FG represents the component of P parallel to DC. This will be a shear force.
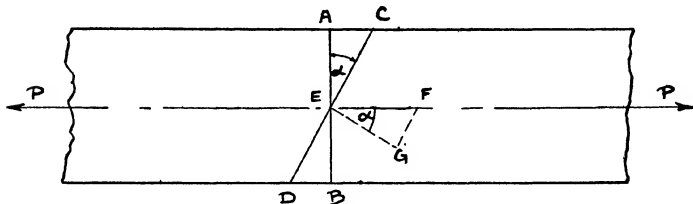
FIG. 35

Let P_r and P_p be the component forces at right-angles and parallel to DC respectively, and f_r and f_p the component stresses at right-angles and parallel to DC respectively.

$$P_r = P \cos \alpha.$$
$$P_p = P \sin \alpha.$$

Area of cross-section $DC = \dfrac{A}{\cos \alpha}$

$$f_r = \frac{\text{Load}}{\text{Area}} = \frac{P \cos \alpha}{\dfrac{A}{\cos \alpha}}$$

$$= \frac{P \cos^2 \alpha}{A} = f \cos^2 \alpha.$$

$$f_p = \frac{P \sin \alpha}{\dfrac{A}{\cos \alpha}}$$

$$= \frac{P}{A} \sin \alpha \cos \alpha = f \sin \alpha \cos \alpha.$$

f_p is a maximum when $\alpha = 45°$; *i.e.*, when $f_p = \dfrac{f}{2}$

f_r and f_p are therefore less than f for any oblique section of a bar of uniform cross-section.

A member may, however, be so shaped that it is weaker on an oblique section than on the section normal to the load, as shown by the following example—

A rectangular tie bar 4 inches deep and 2 inches wide has a rectangular hole 1 inch by $\frac{1}{2}$ inch diagonally through it at 60° to the longitudinal axis, as shown in Fig. 36. Find the normal stress and oblique stresses on a section through the hole, when the bar is subjected to a pull of 40 tons.
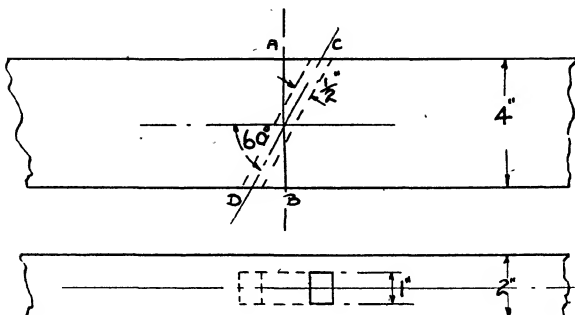
Area of cross-section on line AB $= 4 \times 2 - 1 \times \dfrac{1}{2 \sin 30°}$.

$$= 8 - 1 = 7 \text{ sq. inches.}$$

$$\text{Normal load} = 40 \text{ tons.}$$

$$\text{Normal stress} = \frac{40}{7} = 5 \cdot 7 \text{ tons/sq. inch.}$$

Area of cross-section on line CD $= \dfrac{4 \times 1}{\cos 30°} = 4 \cdot 62$ sq. inches.

$$\text{Shear force on } CD = 40 \sin 30°$$

$$= 20 \text{ tons.}$$

$$\text{Oblique shear stress} = \frac{20}{4 \cdot 62} = 4 \cdot 33 \text{ tons/sq. inch.}$$

$$\text{Tensile force in } CD = 40 \cos 30°$$

$$= 34 \cdot 64 \text{ tons.}$$

$$\text{Oblique tensile stress} = \frac{34 \cdot 64}{4 \cdot 62} = 7 \cdot 49 \text{ tons/sq. inch.}$$

Principal Stresses.

When a body is subjected to more than one type of loading, *e.g.*, tension and shear, the maximum intensity of stress in the body will not be the shear stress or direct tensile stress.

At any point within a body subjected to stress there are three mutually perpendicular planes on each of which the resultant stress is a normal stress. These planes are called Principal Planes, and the resultant normal stresses Principal Stresses.

When, as here, we are only dealing with stresses in two dimensions, the third principal stress will be zero.

Consider a small cube $ABCD$ of material in a body with face area a subjected to tensile stresses f_x and f_y, and shear stress f_s, as shown in Fig. 36A.

Let EC be a principal plane and f_p the principal stress. Thus from definition f_p will be the resultant stress normal to the plane. There will be no tangential stress as f_r in the previous case of oblique stresses.

$$\text{Area } CD = a$$

$$\text{,,} \quad CE = \frac{a}{\sin \alpha}$$

$$\text{,,} \quad ED = \frac{a}{\tan \alpha}$$

Tensile Reaction on CD $= X = f_x a$

Shear   ,,   ,, ,, $= S_x = f_s a$

Tensile   ,,   ,, ED $= Y = f_y \dfrac{a}{\tan \alpha}$

Shear   ,,   ,, ,, $= S_y = f_s \dfrac{a}{\tan \alpha}$

Normal   ,,   ,, EC $= P = f_p \dfrac{a}{\sin \alpha}$

Thus we have ECD in equilibrium under the above forces.

Fig 36a

Resolving horizontally—

$$P \sin \alpha = X + S_y$$
$$f_p a = f_x a + f_s \frac{a}{\tan \alpha}$$
$$f_p = f_x + \frac{f_s}{\tan \alpha}$$
$$f_s \cot \alpha = f_p - f_x \qquad \ldots \qquad \ldots \quad 1.$$

Resolving vertically—

$$N \cos \alpha = Y + S_x$$
$$f_p \frac{a}{\tan \alpha} = \frac{a}{\tan \alpha} + f_s a$$
$$f_p = f_y + f_s \tan \alpha$$
$$f_s \tan \alpha = f_p - f_y \qquad \ldots \qquad \ldots \quad 2.$$

Subtracting 2 from 1

$$f_s (\cot \alpha - \tan \alpha) = f_y - f_x$$
$$2 f_s \cot 2\alpha = f_y - f_x$$
$$\text{Tan } 2\alpha = \frac{2 f_s}{f_y - f_x}$$

This gives two values of α, which give the angle of the two principal planes to AD.

To obtain the principal stresses multiply 1 by 2.

$$f_s{}^2 = (f_p - f_x)(f_p - f_y)$$

$$= f_p{}^2 - f_p(f_x + f_y) + f_x f_y$$

$$f_p{}^2 - f_p(f_x + f_y) + f_x f_y - f_s{}^2 = 0$$

$$\therefore f_p = \frac{f_x + f_y \pm \sqrt{(f_x + f_y)^2 - 4(f_x f_y - f_s{}^2)}}{2}$$

$$f_p = \tfrac{1}{2}\left[(f_x + f_y) \pm \sqrt{(f_x - f_y)^2 + 4 f_s{}^2}\right]$$

This gives two values for the principal stresses, the greater being called the major principal stress and the smaller the minor principal stress.

Bearing Stress.

Consider a bolt of diameter d transmitting a force P to a plate of thickness t (Fig. 37).
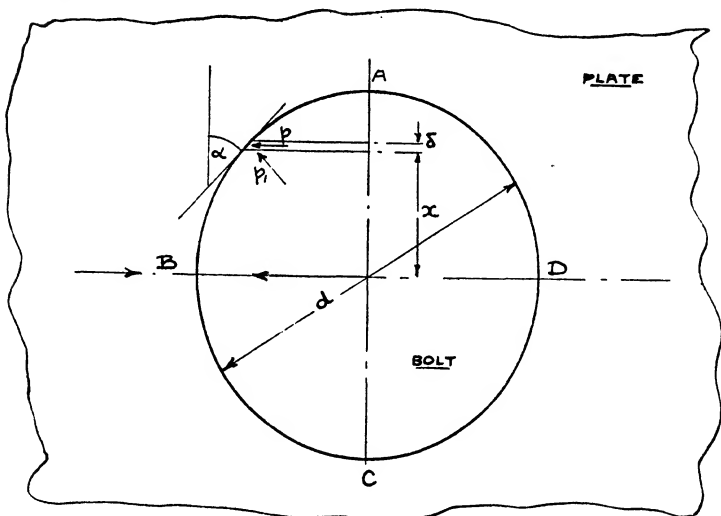
Fig. 37

The force is transmitted by compression along the arc ABC, and tends to crush the bolt and plate.

The stress will vary from zero at A and C to a maximum at B.

Consider a small element of diameter δ along the line AB, distance x above the central line BD, and let α be the angle the tangent of the arc ABC makes with AC at this distance x above BD.

The length of arc subtended by $\delta = \dfrac{\delta}{\cos \alpha}$

and the bearing area $= a = \dfrac{t\delta}{\cos \alpha}$

The force p on this area parallel to BD will be the same proportion of P that δ is of d.

i.e., $p : P :: \delta : d$, or $p = \dfrac{P\delta}{d}$

Let p_1 be this elementary force resolved normal to the area a.

then $p_1 = p \cos \alpha = \dfrac{P\delta}{d} \cos \alpha.$

Let stress at distance x above $BD = f_1$

then $f_1 = \dfrac{p_1}{a} = \dfrac{P\delta \cos \alpha}{\dfrac{dt\,\delta}{\cos \alpha}}$

$= \dfrac{P \cos^2 \alpha}{dt}.$

This will be a maximum at B when $\alpha = 0°$, and its value is $\dfrac{P}{dt}$

Thus maximum bearing stress $= \dfrac{\text{Load}}{\text{Diameter} \times \text{Thickness}}$

<center>EXAMPLES.</center>

1. A tie rod has a diameter of $\frac{1}{2}$ inch. What will be the stress in it when it is subjected to a pull of 15,000 lb. ?

The stress will be tensile.

$$f = \frac{P}{A}$$

$$= \frac{P}{\frac{\pi}{4} d^2}$$

$$= \frac{15000 \times 16}{3 \cdot 14}$$

$$= 76,400 \text{ lb./sq. inch.}$$

2. A $\frac{1}{4}$-inch diameter hole is punched in a steel plate of 0·048 inch thickness.

If the maximum shear stress the plate will withstand is 40 tons/sq. inch, find the load and compressive stress in the punch. Assume the punch is of uniform cross-section.

Area of plate transmitting shear

$\qquad = $ circumference of hole $\times$ thickness of plate.

$$f = \frac{P}{A}$$

Load in punch $\qquad = fA$

$$= 40 \times 2240 \times \pi \times \frac{1}{4} \times 0 \cdot 048$$

$$= 3379 \text{ lb.}$$

Area transmitting compression = cross-sectional area of punch

$$\text{Stress in punch} = \frac{P}{A}$$

$$= \frac{3379}{\frac{\pi}{4} \times \left(\frac{1}{4}\right)^2}$$

$$= 68{,}800 \text{ lb./sq. inch.}$$

3. A steel tube which is 1 inch diameter and 0·025 inch thick is fitted at each end with a socket secured by means of two ¼ diameter taper pins. Find the maximum tensile and bearing stresses in the tube, and the shear stress in the pins, when the tube is subjected to a tensile load of 3,000 lb.

Minimum cross-sectional area of tube

$$= (\text{circumference} - \text{dia. of holes}) \text{ thickness}$$

$$= \left(\pi \times 1 - \frac{1}{2}\right) 0 \cdot 025.$$

$$\text{Maximum tensile stress} = \frac{P}{A}$$

$$= \frac{3000}{\left(\pi - \frac{1}{2}\right) 0 \cdot 025}$$

$$= \frac{120000}{2 \cdot 642}$$

$$= 45{,}400 \text{ lb./sq. inch.}$$

$$\text{Bearing stress} = \frac{P/4}{dt}$$

$$= \frac{3000}{4 \times \frac{1}{4} \times 0 \cdot 025}$$

$$= \frac{3000}{0 \cdot 025}$$

$$= 120{,}000 \text{ lb./sq. inch.}$$

$$\text{Shear stress} = \frac{\text{load on each pin}}{\text{Twice cross-sectional area of pin}}$$

$$= \frac{1500}{2 \left[\frac{\pi}{4} \times \left(\frac{1}{4}\right)^2\right]}$$

$$= \frac{1500 \times 32}{\pi}$$

$$= 15{,}300 \text{ lb./sq. inch.}$$

4. A steel tie rod of 0·023 sq. inches cross-sectional area is 4 feet long. Find its length when subjected to a pull of 3,000 lb.

E for steel $= 30,000,000$ lb./sq. inch.

$$E = \frac{PL}{Ae}$$

$$\therefore \quad e = \frac{PL}{AE}$$

$$= \frac{3000 \times 4 \times 12}{0 \cdot 023 \times 30000000}$$

$$= \frac{48}{230}$$

$$= 0 \cdot 209 \text{ inch.}$$

Length $= 4 \text{ feet} + 0 \cdot 209 \text{ inch}$
                $= 48 \cdot 209 \text{ inches.}$

5. Two flat steel bars 4 inches wide have to be lapped and bolted together in order to form a tension member, to carry a load of 180,000 lb. Find the necessary diameter of the bolt, and the thickness of each bar. Take the permissible tensile and shear stress of the bar and bolt to be 32 tons/sq. inch and 40 tons/sq. inch respectively.

Shear stress $= \dfrac{P}{A}$

$$A = \frac{P}{f}$$

$$A = \frac{\pi}{4}\, d^2 = \frac{180000}{40 \times 2240}$$

$$\therefore \quad d = \sqrt{\frac{180000 \times 4}{40 \times 2240 \times \pi}}$$

$$= \sqrt{2 \cdot 56}$$

$$= 1 \cdot 6 \text{ inches.}$$

Say 1⅝-inch dia. bolt

Tensile stress $= \dfrac{P}{A}$

$$A = \frac{P}{f}$$

$$A = (4 - 1\tfrac{5}{8})\, l = \frac{180000}{32 \times 2240}$$

$$l = \frac{180000}{32 \times 2240 \times 2 \cdot 375}$$

$$= 1 \cdot 05 \text{ inches thickness of bar.}$$

6. A wiring lug is attached to a fork end by means of a $\frac{1}{2}$-inch diameter pin, as shown in Fig. 38.   Under a load of 15,000 lb. the
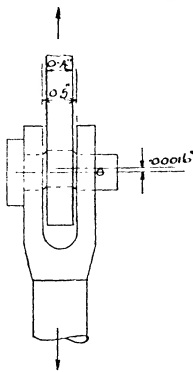
Fig. 38

centre of the pin in the lug is moved 0·00016 inch relative to the centre of the pin in the fork (shown exaggerated in sketch).   Find the modulus of rigidity of the material of the pin.

$$\text{Shear stress} = \frac{P}{A}$$

$$= \frac{15000}{\frac{\pi}{4} \times \frac{1}{4} \times 2}$$

$$= \frac{15000 \times 8}{\pi}$$

$$= 38{,}200 \text{ lb./sq. inch.}$$

$$\text{Shear strain} = \frac{0·00016}{\frac{1}{2}(0·5 - 0·4)} = \frac{0·00016}{0·05}$$

$$= 0·0032 \text{ radians.}$$

$$\text{Modulus of rigidity} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$= \frac{38200}{0·0032}$$

$$= 11{,}900{,}000 \text{ lb./sq. inch.}$$

7. A round bar of steel 2 inches diameter is subjected to a pull of 20 tons.   Find the reduction in diameter.

$$\text{Given } E = 13{,}000 \text{ tons/sq. inch.}$$

$$\text{Poisson's ratio} = \frac{1}{4}$$

$$\text{Tensile stress} = \frac{20}{\frac{\pi}{4} \times 4} = \frac{20}{\pi} \text{ tons/sq. inch.}$$

$$E = \frac{f}{\text{strain}}$$

Longitudinal strain $= \dfrac{f}{E}$

Poisson's ratio $= \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}} = \dfrac{1}{4}$

$\therefore$ Lateral strain $= \dfrac{\text{Longitudinal strain}}{4}$

$\qquad\qquad\quad = \dfrac{f}{4E} = \dfrac{20}{4\pi \times 13000}$

$\qquad\qquad\quad = 0 \cdot 00012.$

Reduction in diameter $= 2 \times$ Lateral strain

$\qquad\qquad\qquad\quad = 2 \times 0 \cdot 00012$

$\qquad\qquad\qquad\quad = 0 \cdot 00024$ inch.

8. A short rectangular block 6 inches wide by 4 inches thick supports a load of 200 tons. Find the maximum shear stress on any oblique section.

Oblique shear stress is maximum on a section at 45° to the line of load.

Max. oblique shear stress $= \dfrac{f}{2}$

$\qquad\qquad\qquad\qquad\; = \dfrac{200}{6 \times 4 \times 2}$

$\qquad\qquad\qquad\qquad\; = 4\dfrac{1}{6}$ tons/sq. inch.

9. A 50 cu. ft. lump of metal is dropped into the sea to a depth of one mile. Find the reduction in volume in cubic inches due to the pressure of the sea. Given: Bulk modulus of metal = 20,000,000 lb./sq. inch. Weight of sea-water = 64 lb./cu. foot.

Stress $=$ pressure $=$ head $\times$ wt. per unit vol. of sea-water

$\qquad\qquad\qquad\quad = 5280 \times 64$ lb./sq. foot

$\qquad\qquad\qquad\quad = \dfrac{5280 \times 64}{144} \doteq 2347$ lb./sq. inch.

$K = \dfrac{\text{Stress}}{\text{Vol. strain}}$

$\therefore$ Volumetric strain $= \dfrac{\text{Stress}}{K}$

$\qquad\qquad\qquad\qquad = \dfrac{2347}{20000000}$

$\qquad\qquad\qquad\qquad = 0 \cdot 0001173.$

Reduction in volume $=$ Vol. strain $\times$ Original vol.

$\qquad\qquad\qquad\qquad = 0 \cdot 0001173 \times 50 \times 1728$

$\qquad\qquad\qquad\qquad = 10 \cdot 14$ cu. inches.

10. A $\frac{3}{4}$ inch diameter bolt is subjected to a pull of 10 tons, and at the same time to a shearing force of 3 tons, such that it is in single shear. Find the maximum intensity of stress.

$$f_s = \frac{P_s}{A} = \frac{3 \times 4 \times 16}{\pi \times 9}$$

$$= 6 \cdot 8 \text{ tons/sq. inch.}$$

$$f_x = \frac{P_t}{A} = \frac{2 \times 4 \times 16}{\pi \times 9}$$

$$= 4 \cdot 54 \text{ tons/sq. inch.}$$

$$f_p = \frac{4 \cdot 54 \pm \sqrt{4 \cdot 54^2 + 4 \times 6 \cdot 8^2}}{2}$$

$$f_{max} = \frac{4 \cdot 54 + \sqrt{205 \cdot 6}}{2}$$

$$= 9 \cdot 43 \text{ tons/sq. inch.}$$

MECHANICAL PROPERTIES AND TESTING OF MATERIALS

Mechanical Properties of Materials.

In addition to the strength and elasticity, there are several other mechanical properties on which the suitability of a material for any particular job depends.

These properties may be defined as follows—

Ductility.—The ability to withstand a large amount of deformation without fracture, more strictly applied to tensile deformation. That is, a material is ductile if it can be drawn out by tension to a smaller cross-section, such as in wire drawing.

Malleability.—This is very similar to ductility, and is the property which allows a material to be permanently deformed without fracture when beaten or rolled.

Toughness.—The ability to resist fracture when subjected to a sudden blow, bending or twist.

Brittleness.—Lack of ductility, malleability or toughness.

Hardness.—The property of a material to resist denting or scratching by another material. A hard material will resist wear.

Tensile Test.

In order to ascertain the mechanical properties of materials, numerous methods of testing are resorted to, the most widely used being the tensile test.

In this test a piece of metal of known dimensions is subjected to a gradually increasing load until it breaks.
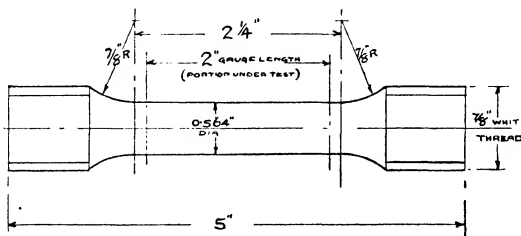
FIG. 39

A standard test piece for steel that has been adopted in the British Standard Specifications is illustrated in Fig. 39. This standard piece has a cross-sectional area of ¼ sq. inch, so that the tensile stress may be easily found by multiplying the tensile load by 4. The elongation being measured on the gauge-length, the strain will equal this elongation divided by 2.

This is not by any means the only type of test piece used for a tensile test, but in every case the portion under test must be of uniform and minimum cross-section. There must be no sudden changes of area or fracture will often take place there instead of in the smaller portion under test.

There are several types of tensile testing machines, the most usual being illustrated diagrammatically in Fig. 40.
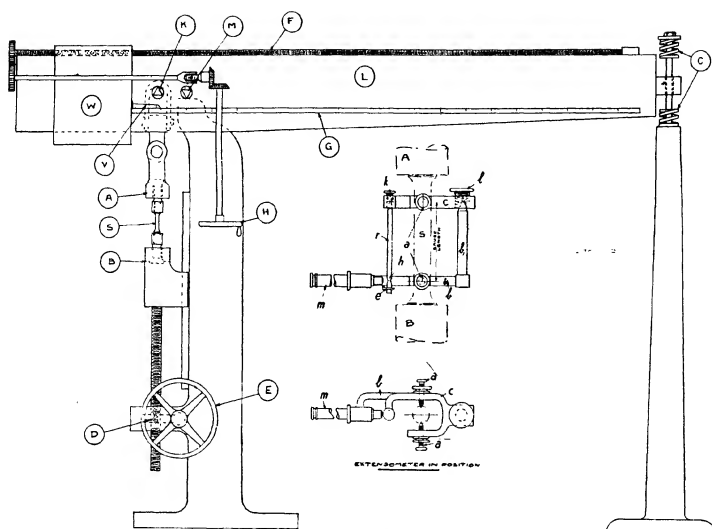
FIG. 40

The specimen S is gripped at A and B, the method depending on the type of specimen used.   The grip A is connected through a universal joint to the knife-edge K on the lever L.   This lever is exactly balanced about the knife-edge M when the jockey weight W is in the zero position —*i.e.*, when the zero on the vernier V attached to the weight W coincides with the zero on the graduated scale G attached to the beam.

The test is started with no load on the specimen, and the lever floating between the spring stops C.   This is done by having the jockey weight at zero and moving the grip B up or down by means of the worm gear D, until the lever is floating.   The worm gear D may be worked by the hand-wheel E or an electric motor.

In order to measure the elongation of the specimen between the gauge points, an instrument called an extensometer is attached. There are several types of extensometer, all having for their function the magnification of the extension.   The Ewing type is illustrated diagrammatically in the figure.   The two clips b and c are attached to the test piece S by the points of the set-screws a at the distance of the gauge length apart, the points of attachment having been previously marked in a jig with a centre punch.   The clip b has a spherical-ended projection b^1, kept in engagement by means of a spring (not shown) with a conical hole in c.   A rod r engages in the same manner with the other end of c, and is free to slide in the guide e on b.   At h there is a hole in the rod crossed by a hair on which is sighted the microscope m attached to b.   As the specimen extends the rounded ends of b^1 and r act as fulcrums, and the hair h is raised relative to b, an amount equal to twice the extension.   This displacement is measured by means of a micrometer scale in the eyepiece of the microscope.   When the extension is such that the hair is displaced off the scale, it may be brought back by turning the screw l, in which case the number of turns should be recorded, and for each turn the pitch of the screw l added to the extension.

It is advisable to remove the extensometer before the specimen breaks.

When the extensometer is in position and adjusted to the zero reading on the scale by means of the screw k, a load is slowly applied to the specimen, by turning the hand-wheel H, which through suitable gearing turns the screw F, which in turn moves the jockey weight W along the lever by means of a nut fixed to W.

The lever will drop towards the lower stop C, due to the specimen stretching, and should be brought back and kept in an approximately horizontal position by lowering the bottom grip B with the mechanism D.

If a series of loads are measured on the scale G and the corresponding extensions on the extensometer, a graph of the results may be plotted.

It is usual to plot stress as ordinates and strain as abscissæ, the stress being found by dividing the loads by the original cross-sectional area, and the strain by dividing the extensions by the original length. Such a graph is called a stress-strain graph.

A typical stress-strain graph for mild steel is given in Fig. 41. Graph (a) is drawn by a special automatic recording apparatus which records right up to the break, but as this does not show clearly what happens at the important portion AB, graph (b) is drawn to a larger strain scale from extensometer readings.

FIG. 41

Considering graph (b), it will be noticed that the portion of the curve AC is a straight line—that is, up to the point C stress is proportional to strain, and the material obeys Hooke's Law.

Hooke's Law states that within the elastic limit the strain is proportional to the stress producing it.

The **Elastic Limit** may be defined as the limit of stress within which the strain entirely disappears when the stress is removed—*i.e.*, the highest stress at which the extensometer reading will return to zero when the load is removed.

The **Limit of Proportionality** is the limit of stress within which stress is proportional to strain. It does not always coincide with the elastic limit, but is very nearly the same for mild steel. In the practical use of materials this limit is of the first importance. Most of the strength formulæ are based on the assumption that stress is proportional to strain, and of course the Modulus of Elasticity only applies within this limit.

The true elastic limit and limit of proportionality are very difficult to obtain, as in very accurate experiments it is found that the curve
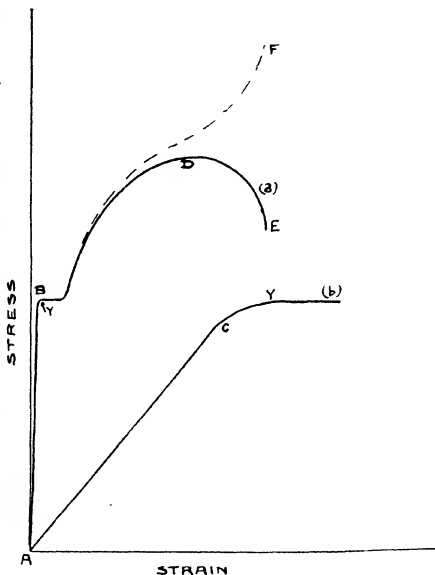
slightly leaves the line of proportionality at much lower stresses than is usually revealed. As in use the material may not be appreciably strained above the elastic limit or limit of proportionality, it is necessary to have some definite limit of stress which will meet this requirement and yet may be easily measured. It is for this reason that the Yield Stress and Proof Stress are used.

Yield Point.—At the point Y on the graphs it will be noticed that there is an increase of strain without an increase of stress. This is called the Yield Point, and the corresponding stress the Yield Stress. It is well defined in wrought iron and low carbon steels, and for these materials is usually the criterion of the maximum allowable stress which may be applied to the material in an aeroplane structure. For most non-ferrous materials and alloy steels where the yield point is not well defined, a limit called the Proof Stress is applied.

Fig. 42 (a) shows graphs for a nickel-chromium steel in which the yield point is not apparent, and for duralumin which has several yield points well above the limit of proportionality.

Proof Stress may be defined as the tensile stress which, when applied for fifteen seconds and removed, produces a permanent set of a specified amount (usually 0·1 per cent.) of the original length—*i.e.*, it is the
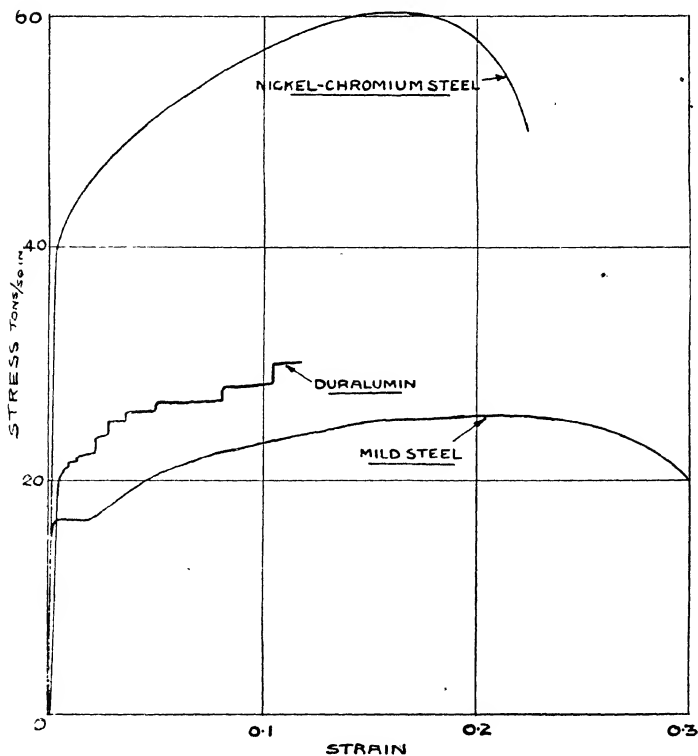
Fig. 42a

stress at which the stress-strain curve departs by a specified percentage from the Limit of Proportionality, as shown in Fig. 42 (*b*).

In the British Standard Specifications 0·1 per cent. Proof Stress is defined as " the maximum load per square inch which when applied to a tensile test piece for fifteen seconds and removed produces a permanent extension of not more than 0·1 per cent. of the gauge length."
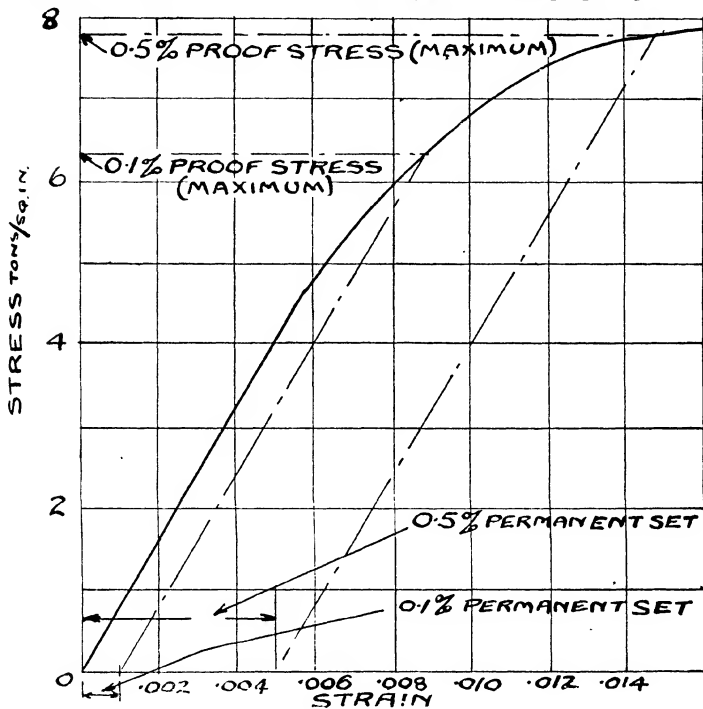
FIG. 42 (*b*)

Ultimate Strength.—At the point *D* on graph (*a*), Fig. 41, the curve has reached its highest ordinate, and this is called the Ultimate Stress or Ultimate Strength of the material. For all practical purposes this could·be considered as the maximum stress, as after this point is reached the specimen will still go on stretching when some of the load has been removed until it breaks at the point *E*, due to the local contraction of area and stretching at the part where it is about to break as shown in Fig. 43.
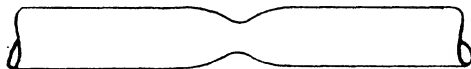
FIG. 43

Actually *E* is not the real breaking stress, for if the load is divided by the reduced area instead of the original area, the graph will follow the dotted line, and show its maximum stress at *F*.

Phenomenon of Strain.—When a specimen of ductile material, such as mild steel is stressed within the elastic limit, elongation takes place due to the molecules being forced further apart. When this stress is removed there will no longer be any force increasing their relative distances apart, and they will return to their original distances. There having been no displacement of the molecules, there will be no permanent set.

If the specimen is stressed beyond the elastic limit, not only will the molecular distances have been increased, but the crystals will have slipped along their planes of cleavage, thus displacing the molecules. When now the stress is removed, the molecular distances will be decreased as before, but the displaced molecules will remain in their new positions. Thus the specimen will return partly to its original position, but have a permanent set due to the displaced molecules. This is represented diagrammatically in Fig. 29.

Slip Bands.—This phenomenon may be observed by stretching a polished specimen, and watching its surface under the microscope. Up to the elastic limit no change will be apparent. As soon as the yield point is reached, dark lines appear on the polished surface of the crystals, increasing in numbers as the stress is increased. These dark lines, called Slip Bands, are really minute steps formed on the surface, due to the crystals slipping along their planes of cleavage. They will remain after the stress has been removed.

The slip does not continue along one set of planes until fracture occurs, but is arrested due to a layer of atoms, which have been displaced by the slip, resisting the motion along the cleavage plane and finally stopping the sliding. As the stress is increased, slip takes place along other planes, and crystals which offered a bigger resistance to slip than the previous planes.

The result will be that the material is now offering a bigger resistance to deformation. If the stress is removed, it will be found that a bigger stress has to be applied than previously, before the elastic limit is reached.

Overstrain.—Fig. 44 (*a*) is a stress-strain graph of a piece of copper which has been stressed above the yield point, and the stress removed at *a*, *b*, *c*, and restressed. It will be noticed that each time the stress
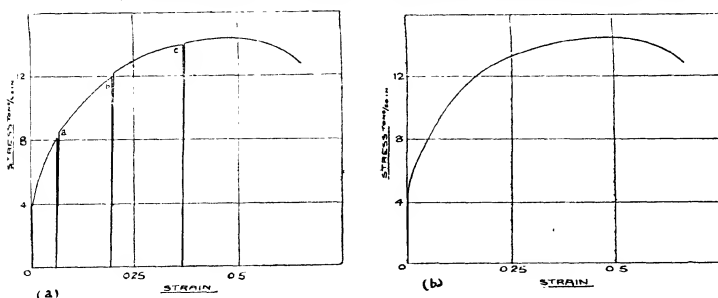
Fig. 44

has been removed the yield point has been increased to a little greater than the stress previous to removal. It will be seen that the complete diagram is very similar to that of the continuously loaded specimen given in Fig. 44 (*b*).

A material which has been thus stressed past the yield point and unloaded is said to be overstrained.

Annealing will destroy the effect of overstrain.

Working Stress and Factor of Safety.

In general engineering, when designing a structure the worst possible loads that can come on it are estimated. These are called the Working Loads, and the corresponding stress the Working Stress. For safety this working stress must be within the limit of proportionality; also in many cases the stress in a member can only be calculated as long as the stress is proportional to the strain.

The working stress is usually determined by dividing the ultimate stress by a figure called the Factor of Safety. This factor of safety depends on the reliability of the material, whether the load is fixed or fluctuating, the degree of accuracy with which the working load can be estimated, and the deterioration which is likely to occur in use.

$$\text{Factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Working Stress}} = \frac{\text{Ultimate Load}}{\text{Working Load}}$$

Load Factor.—It is impracticable in aeronautical engineering to estimate the loads under all the conditions of flight. The loads which come on the aeroplane under certain extreme conditions of steady flight and landing are found, and these are multiplied by a number called a Load Factor, and the component designed to withstand the resulting load.

$$\text{Thus, Load Factor} = \frac{\text{Load part is designed to withstand}}{\text{Load in steady flight}}$$

There are two types of load factor used, viz.,—

(1) *Ultimate Factor.* This is intended to allow for twice the loads expected during manœuvres appropriate to the type of aeroplane. The structure must be designed so that it will not collapse before withstanding the product of the steady flight load and the Ultimate Factor.

(2) *Proof Factor.* This is 75 per cent. of the ultimate factor, and is to allow for the structure being airworthy, *i.e.*, without appreciable permanent set, under $1\frac{1}{2}$ times the maximum loads expected under manœuvres appropriate to the type.

It will be noticed that a margin of safety is left to allow for loads above those normally expected, but which may be put on by the clumsy pilot, or in emergency.

EXAMPLE.

A tensile member has to carry a load of 5,000 lbs. in a condition of steady flight for which the Ultimate Factor is 5, and 4,000 lbs in another condition of steady flight for which the Ultimate Factor is 7.

Find the cross-sectional area of the member; given ultimate stress 52 tons/sq. in. proof stress 36 tons/sq. in.

Load corresponding to Ultimate Factor

1st Case = 5000 × 5 = 25,000 lb.
2nd Case = 4000 × 7 = 28,000 lb.

The second case is greater and so determines the maximum load the member must withstand.

Load corresponding to Proof Factor in 2nd case = 28,000 × 0·75
= 21,000 lb.

The member must not exceed the proof or yield stress under this load $f = P/A$

$$A = P/f = \frac{21000}{36 \times 2240}$$

= 0·26 sq. inch

For the load corresponding to the Ultimate Factor, use ultimate stress

$$A = P/f = \frac{28000}{52 \times 2240}$$

= 0·24 sq. inch.

The larger will be the area required *i.e.*, 0·26 sq. inch.

Measurement of Ductility.

Besides measuring the strength and elasticity of a material, the tensile test is also used to measure its ductility.

The standard of measurement is the elongation per cent. of the original length, measured after fracture on a standard test piece of 0·564 inch diameter on a gauge length of 2 inches.

The higher the percentage elongation, the greater the ductility.

It is necessary to specify a standard gauge length of 2 inches, over which the elongation must be measured, as due to the proportionally greater elongation that takes place about the point of fracture ; a short gauge length would show a larger percentage elongation than a long one.

For sheet metal it has been found better to substitute a bend test, as the percentage elongation has been found to decrease with thickness.

Bend Tests.

There are two types of bend tests for sheet metal—the Close Bend and the Reverse Bend.

In the Close Bend test, the sheet must stand, without signs of failure, being bent over and closed down on another sheet of the same thickness or, in some less ductile materials, twice the thickness.   The strip of metal should have its edges smoothed and rounded off before testing, as rough edges will start cracks and make the test fail.

The Reverse Bend test is carried out by gripping one end of a strip of the material in a vice, which has the edges of the jaws rounded to a radius which varies from one to three times the thickness of the sheet, according as to whether the tensile strength is low or high.   The projecting end of the strip is then bent at right angles to the fixed part, first to one side, then to the other through 180° until it breaks.

The smaller the radius over which it will bend, and the greater the number of reversals, the greater the ductility.   Most specifications for sheet metal call for these bend tests, stating the radius, and the number of reversals the material must stand, without sign of fracture.

Specifications for bar material often call for a bend test, in which the bar has to withstand bending through a certain angle, over a radius of specified proportions to the diameter of the bar.

Hardness Test.

This test is not only used to test the hardness of a material, but also to investigate its quality, and the effects of heat treatment where it is not possible to make a tensile test.

Brinell Test.—The most common hardness test is the Brinell, and it will be sufficient to describe this one.

It is performed by pressing a hardened steel ball into the material at a definite load for fifteen seconds.

The Brinell Hardness Number, which is a standard by which hardness can be compared, is obtained by dividing the load on the ball in kilogrammes by the surface area of the spherical impression measured in square millimetres—

$$i.e., \text{Hardness Number} = \frac{L}{\frac{\pi D}{2}\left(D - \sqrt{D^2 - d^2}\right)}$$

where L = Load in kilogrammes,

D = Diameter of ball, millimetres,

d = Diameter of impressions, millimetres.

As it has been found that the hardness number varies with load and ball diameter, it has been found necessary for purposes of comparison

to standardize the test. For steel and materials of similar hardness, the load used is 3,000 kg. and the ball diameter 10 mm. For copper alloys $L/D^2 = 10$—*i.e.*, 1,000 kg. for the same sized ball ; for copper $L/D^2 = 5$; and for lead and tin $L/D^2 = 1$.

The specimen must be tested on a flat portion such that the flat width each side of the impression is not less than $2d$; the surface scale must be removed and the thickness of material must not be less than 0·32 inch for a soft material, having a hardness number of 100, decreasing to 0·063 for a very hard material with a hardness number of 500.

Referring to the tempering chart, Fig. 28, it will be seen that the degree of tempering may be readily obtained by testing the hardness of the finished product and finding the corresponding hardness number on the chart ; *e.g.*, if the hardness number so obtained for this steel is 400, it may be correctly assumed that the reheating temperature was 405° C., the maximum stress 89 tons/sq. inch, elongation per cent. 12·5, etc.

There seems to be an approximate relationship between the hardness numbers and the ultimate tensile strength of steels. As a rough estimate the ratio

$$\frac{\text{Ultimate tensile stress tons/sq. inch}}{\text{Hardness Number}} = 0\cdot22,$$

and for light alloys, such as duralumin,

$$\frac{\text{Ultimate tensile stress tons/sq. inch} + 1}{\text{Hardness Number}} = 0\cdot25.$$

Vickers Pyramid Hardness Test.

Another test for hardness on the same principle as the Brinell test, in that the hardness is calculated in terms of $\dfrac{\text{Load}}{\text{Impressed area,}}$ is the Vickers. This is performed by pressing a small 136° diamond pyramid into the specimen at a load which may be varied from 1 to 120 kilograms, though the load for the majority of specimens is 30 kilograms.

The advantages of this method are that the impression being very small little damage is done to the material, it may be applied to very thin sheet and small specimens, and the load is applied and removed automatically after a predetermined interval.

The Brinell and Vickers hardness numbers are the same up to a little over 200, but diverge with increasing hardness. The Vickers hardness numbers are called Vickers Pyramid Numerals (V.P.N.).

Impact or Notched Bar Test.

Most of the British Standard and D.T.D. Specifications for steels require that the steel shall give a specified Izod value.

This Izod value is obtained from the result of a test called the Impact or Notched Bar Test.

Although a brittle material gives a very low Izod value, the test does not really measure a metal's resistance to impact or shock. The test is used to find whether the metal has been heat-treated satisfactorily.

Two specimens of the same material, if heat-treated differently, will often give very similar results on the tensile test, but the Izod values will be different ; a relatively high value indicating satisfactory heat treatment.

After the best heat treatment the Izod values will vary greatly for different kinds of steels, but will be high for the particular type of steel treated. It is therefore impossible to compare two steels, such as a carbon steel and a nickel steel, upon the basis of their Izod values.

The test may also be used to indicate the capacity of a material to resist the spreading and formation of cracks. Defects such as minute flaws, hair cracks and inclusions may exist in a finished member, which will tend to spread in service under repeated forces of an impulsive character. Some materials resist this tendency more than others ; those resisting it have a high Izod value.
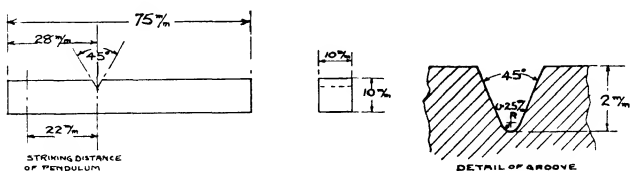
FIG. 45
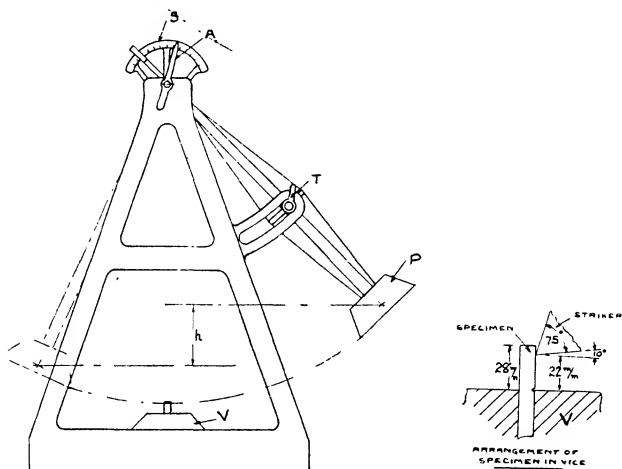
FIG. 46

The standard test piece used is shown in Fig. 45. It is held upright in a rigid vice V at the bottom of the testing machine shown diagrammatically in Fig. 46. The weighted pendulum P is released by the trigger T, which swings down, fractures the specimen and passes onward. The angle it reaches on the other side of the specimen is recorded on the scale S by the sliding pointer A, which is left behind on the return stroke.

The difference in height (h ft.) of the C.G. of the pendulum at the commencement of its swing and at the end of its swing, multiplied by the weight of the pendulum (W lb.), gives the energy expended in fracturing the specimen, plus the energy expended in friction. In order to obtain the energy expended in friction, the test is carried out without a specimen, and the difference in height (h^1 ft.) multiplied by the weight (W lb.) gives this energy.

The Izod value, which is the energy absorbed in fracturing the specimen, therefore equals

$$hW - h^1W$$
$$= W (h - h^1) \text{ ft. lb.}$$

As the pendulum always starts from the same position, the scale S is graduated to record this Izod value directly in ft. lb.

Fatigue of Metals.

With the advent of high-speed machinery, it was discovered that materials fracture under stresses considerably lower than the ultimate stress obtained by the normal static test if these stresses are repeated many times, and at still lower stresses if the stress is repeatedly reversed —e.g., changes from tension to compression. This behaviour of metals when subjected to fluctuating stress is known as Fatigue. It occurs in most engine parts and those parts of the aeroplane and other structures which are subject to vibration. Members such as longerons, where the stress only occasionally varies from tension to compression, need not be considered under this heading.

The following terms are used in connection with the fatigue of metals—

Varying Stresses are stresses of the same kind, fluctuating between a maximum and a minimum value—e.g., 20 tons/sq. inch tension to 10 tons/sq. inch tension.

Alternating Stresses are those in which the fluctuations are from stress in one direction to stress in the opposite direction—e.g., tension to compression, or clockwise torsion to anti-clockwise torsion.

Reversed Stresses are those in which the stresses fluctuate from one direction to the same value in the opposite direction—e.g., 10 tons/sq. inch tension to 10 tons/sq. inch compression.

(*Note.*—When considering fatigue compressive stress is taken as a negative tensile stress.)

Range of Stress is the algebraical difference between the maximum and minimum stresses.

Endurance is the number of fluctuations required to produce failure.

Endurance Limit is the highest stress which will give infinite reversals without failure.

Fatigue Range or **Limiting Range of Stress** is the maximum range of stress which will give infinite reversals without failure.

Fatigue Tests.

The commonest method of determining the fatigue range is the Wohler reversed bending test. In this test the specimen shown in detail in Fig. 47 is held at the large end in a split collett on the end of a shaft rotated by an electric motor. The free end of the specimen carries a load applied constantly in one direction. Fig. 48 shows diagrammatically a modern arrangement of this testing machine.

Under this arrangement the specimen is a cantilever, having tension on the bottom half, and an equal compression on the top half. (See " Theory of Beams," Chapter XI.) If the specimen is rotated through half a revolution, the top half, which was before in compression, is now the bottom half, and in tension.

To perform the test a load P lb. is put on the end of the specimen by moving the jockey weight W along the lever L until the load on the specimen S is such that the maximum stress, which will be at X, the end of the parallel portion, is about two-thirds the ultimate statical stress of the material found from the ordinary tensile test.
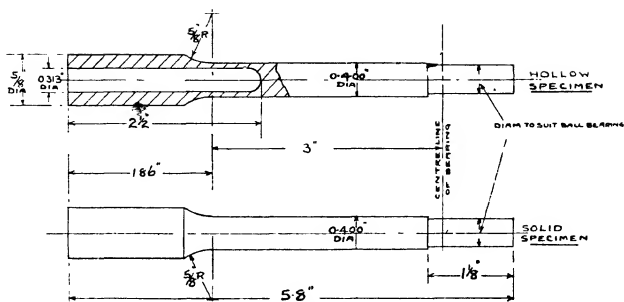
FIG. 47
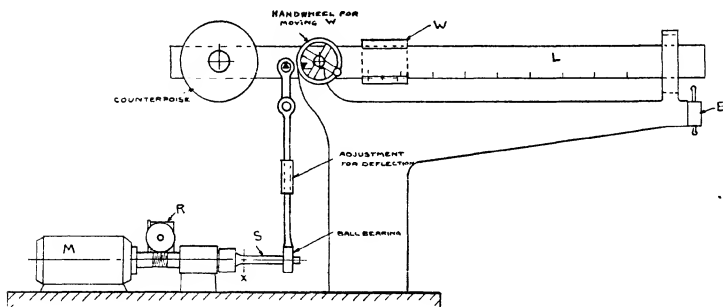
FIG. 48

This stress for the specimens shown in Fig. 47 will equal—

477 P lb. per sq. inch for solid specimens ;

764 P lb. per sq. inch for hollow specimens.

The range of stress will equal twice the above figures.

The electric motor M is run and the specimen thus revolved until failure takes place, when the motor is automatically stopped by the mercury switch E, on which that end of the lever drops. The number of reversals of stress or endurance is found by recording the revolutions of the specimen on a suitable revolution counter R.

A second specimen of the same material is now placed in the machine and subjected to a slightly smaller stress, and the endurance found as before. The test is repeated with diminishing stresses until the specimen stands up to twelve million reversals for the solid specimen, or six million reversals for the hollow specimen.

A graph of the results should be plotted, having ordinates to represent either the maximum stress or the range of stress and abscissa to represent the endurance. Such a graph is usually called a S/N curve. Results of a test in medium carbon steel, having an ultimate tensile strength of 35 tons/sq. inch, are shown in the table on p. 57, and the corresponding S/N curve in Fig. 49. A solid specimen was used.

It will be noticed that as the stress decreases the endurance increases, until the curve tends to become horizontal. At a stress of 16·6 tons/sq. inch, where the curve has become apparently horizontal, the point is

reached where the endurance has become indefinitely large. The stress at this point is the endurance limit, and the range of stress the fatigue range. In this case the endurance limit is 16·6 tons/sq. inch, and the fatigue range 33·2 tons/sq. inch.

A hollow specimen will give a curve which will become horizontal at a lower endurance, but the endurance limit will be practically identical.

It may be seen from the results that with a motor which usually runs at about 2,000 r.p.m. it must take several days to complete one
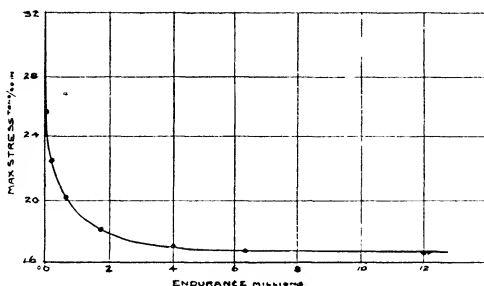
FIG. 49

Maximum Stress. Tons/sq. inch.	Range of Stress. Tons/sq. inch.	Endurance. Millions.
± 25·6	51·2	0·054
± 22·4	44·8	0·2
± 20·2	40·4	0·602
± 18·1	36·2	1·71
± 17·0	34·0	4·01
± 16·6	33·2	12 (unbroken)
± 16·7	33·4	6·32

test. For this reason alone it would not be practicable to use this test for routine testing. It is not, therefore, within the scope of this book to describe other types of fatigue tests and testing machines, but to use the example taken to help make clear the practical significance of the results obtained by many investigators.

For those readers who would like to study this subject further, we would recommend them to read " Fatigue of Metals," by H. S. Gough.

General Practical Conclusions.

Endurance Limit and Fatigue Range.

For ferrous metals the endurance limit or fatigue range for all practical requirements may be taken as the maximum stress or range of stress respectively, at which the specimen remains unbroken after enduring 10,000,000 reversals.

For a hollow rotating cantilever specimen, this may be reduced to 6,000,000 reversals.

For non-ferrous metals the number of reversals required to obtain

the endurance limit is much greater.   There are not, however, sufficient
data available to say what this number should be, but for duralumin
it has been found that the S/N curve did not become horizontal before
10^9 reversals were reached.

Surface of Specimen.

It has been found that the finished surface of the specimen affects
the endurance limit. A roughly turned specimen only shows an
endurance limit of about 91 per cent. of a ground specimen, and about
84 per cent. of a polished specimen.   A small scratch or slight corrosion
will reduce the endurance limit very considerably.

Sondevicker found that a polished specimen lost 40 per cent. of its
endurance limit when scratched to a depth of 0·003 inch.

The foregoing shows the necessity of having a good finish on those
parts of an engine or structure which are subject to fatigue.   Those
readers who have seen the components of an aero engine may have
wondered why so much trouble is taken to give them a polished finish.
This polish not only gives them a better resistance against fatigue,
but also brings to view any defects such as hair cracks, which would
reduce the fatigue range.

Relation between Endurance Limit and Ultimate Stress.

As the result of numerous experiments, it has been found that there
is an approximate relationship between the endurance limit and the
ultimate stress of ferrous metals.

The average value of $\dfrac{\text{endurance limit}}{\text{ultimate stress}}$ is 0·46 for reversed bending.

The endurance limit obtained in this way will only be approximate,
but the maximum error involved will not in any case exceed ± 10 per
cent.

This estimate cannot be used for non-ferrous metals, neither can
it be used to obtain the fatigue range, except for reversed stresses.
For stresses that alternate between different maximum and minimum
values there is no general relationship for the range of stress.

Theory of Fatigue.

It has been found that fatigue failure is due to the development of
tiny cracks which spread across the material.   It is thought that
these cracks originate at the slip-bands, which occur at points of
maximum stress; but whether this is true, and, if so, why the slip-bands,
which are strengthened after slipping under static stress, should be
weakened into cracks under alternating stress is not known.

Several theories have been advanced, and although many have much
in common, there are certain unproved assumptions made.   Until
further investigation brings more knowledge, it will not be known
which, if any, of the present theories is correct.

The theories are summarized in " The Fatigue of Metals," by Gough,
already referred to.

<div align="center">EXAMPLES.</div>

1. The following table gives the result of a tensile test on mild steel.
At a load of 4·3 tons the extensometer was removed.   A standard
specimen of $\frac{1}{4}$ sq. inch cross-sectional area was used, and the extensions
were measured on a gauge length of 2 inches.   The maximum load was
6·35 tons, and the total extension of the specimen at fracture between
the gauge points was 0·64 inch.

Plot stress-strain graphs—(a) using a large strain scale to show
strains up to 0·0015, (b) using a small strain scale, showing strains up

to fracture. On graph (*b*) show by a dotted line the form you would expect the curve to take after the load of 4·3 tons was reached.

Mark on the graphs the yield stress, the ultimate stress and the approximate elastic limit.

Load—tons	0·5	1·0	2·0	3·0	3·7
Extension—inches ...	0·00034	0·000605	0·00120	0·00184	0·00225

Load—tons	4·0	4·1	4·2	4·22	4·3
Extension—inches ...	0·00246	0·00256	0·00270	0·0324	0·0342

$$\text{Stress} = \frac{\text{Load}}{\frac{1}{4}} \qquad \text{Strain} = \frac{\text{Extension}}{2}$$

Stress—Tons/sq. inch	2·0	4·0	8·0	12·0	14·8
Strain	0·00017	0·000302	0·00060	0·00092	0·001125

Stress—Tons/sq. inch	16·0	16·4	16·8	16·88	17·2
Strain	0·00123	0·00128	0·00135	0·0162	0·0171
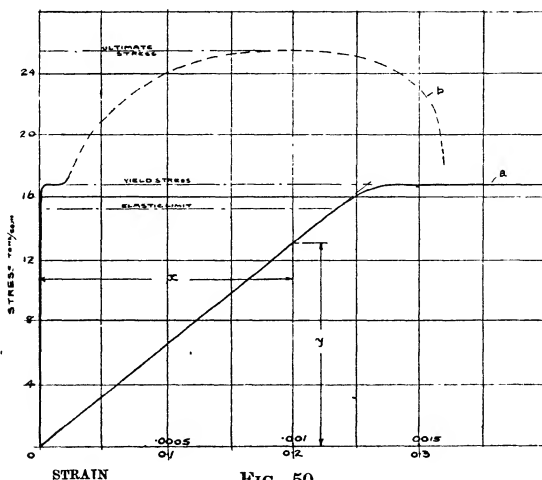
Fig. 50

The graphs are shown in Fig. 50.

2. Using the results in Example 1, find—

      (*a*) The elongation per cent.

      (*b*) Young's Modulus.

(a) Total extension = 0·64 in.

Original length = 2 inches.

Therefore elongation per cent. $= \dfrac{0·64 \times 100}{2} = \underline{32.}$

(b) Young's Modulus $(E) = \dfrac{\text{Stress}}{\text{Strain}}$

= Slope of graph within the limit of proportionality.

Referring to graph—

$$E = \frac{y}{x} = \frac{13·1}{·001} = \underline{\underline{13,100 \text{ tons/sq. inch.}}}$$

3. A certain steel is tested in a Brinell hardness testing machine. The load used was 3,000 kg. and the ball diameter 10 mm. The diameter of the impression was found to be 3·2 mm. Find the Brinell hardness number for this steel.

What would you expect to be the approximate ultimate tensile strength of the steel ?

Brinell hardness number $= \dfrac{L}{\dfrac{\pi}{2} D \left(D - \sqrt{D^2 - d^2}\right)}$

$$= \frac{3000}{\dfrac{\pi}{2} \times 10 \left(10 - \sqrt{10^2 - 3·2^2}\right)}$$

$$= \frac{3000}{\pi \times 5 \times 0·526}$$

$$= \underline{\underline{363.}}$$

Approximate ultimate strength

= Hardness number × 0·22

= 363 × 0·22

= $\underline{\underline{80 \text{ tons/sq. inch.}}}$

4. If the steel in Example 3 was the nickel-chromium steel for which the tempering chart is given in Fig. 28, write down—

(a) The reheating temperature ;

(b) The Yield Stress ;

(c) The Ultimate Stress ;

(d) The Izod value that would be expected, if the heat treatment was satisfactorily carried out.

From chart, Fig. 28—

(a) 470° C.

(b) 72·5 tons/sq. inch.

(c) 79 tons/sq. inch.

(d) 24·5 ft. lb.

5. What approximate endurance limit would you expect from a specimen of 0·21 per cent. carbon steel which had a static ultimate tensile strength of 31·6 tons/sq. inch ?

$$\frac{\text{Endurance Limit}}{\text{Ultimate stress}} = 0\text{·}46 \text{ approx.}$$

∴ Approximate endurance limit
$$= \text{Ultimate Stress} \times 0\text{·}46$$
$$= 31\text{·}6 \times 0\text{·}46$$
$$= 14\text{·}5 \text{ tons/sq. inch.}$$

Torsion Test.

See Appendix II.

CHAPTER VII.

RIVETED JOINTS—WIRING LUGS—THIN CYLINDERS

Riveted Joints.

The two usual methods of making a riveted joint for plates in tension are by means of a lap joint or butt joint.

In a lap joint the two plates to be joined overlap each other, and the rivet or rivets are put through the plates at the overlapped portion as in Fig. 51 (a). The disadvantage of this type is that the pull on the two plates being out of line, there will be a couple, tending to distort the plates until the pull is in one line, as shown in Fig. 51 (b).

(b)

(a)
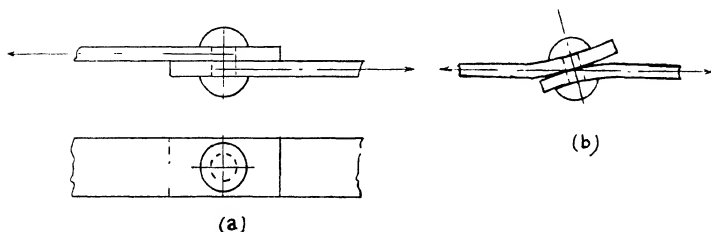
FIG. 51

In a butt joint the two plates are butted together, and a short cover plate is placed on each side. The joint is made by riveting to the cover plates on each side of the butt, as shown in Fig. 52.
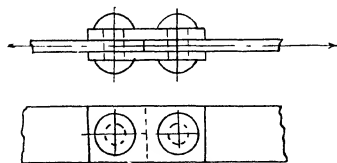
FIG. 52

It will be seen that in this joint the pull is central, and therefore there will be no tendency to distortion.

Types of Failure.—A riveted joint may fail in any of the following ways—

(1) By tearing the plate—Fig. 53 (a).

(2) By shearing the rivets—Fig. 53 (b).

(3) By crushing the plate or rivet—Fig. 53 (c).

(4) By bursting through the plate—Fig. 53 (d).

Number (4) is allowed for by making the distance between the centre of the rivet and the edge of the plate $1\frac{1}{2} \times$ diameter.

LAP
SINGLE SHEAR   DOUBLE SHEAR

(a)   (b)   (c)   (d)
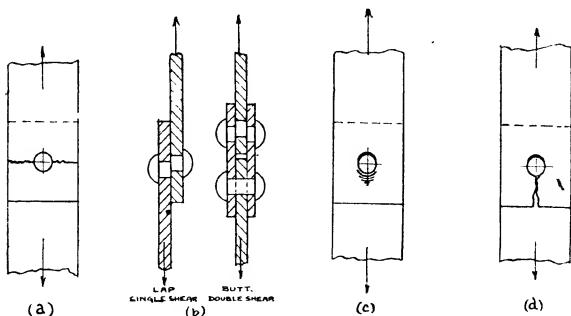
Fig. 53

Strength of Riveted Joints.

In order that there may be no unnecessary material used, and that the joint may be as efficient as possible, it should be arranged that the rivet and the plate will have the same strength. That is, the resistance to tearing of the plate, at the rivet hole where it is weakest, should equal the resistance to shear of the rivet.

The resistance of a body is equal to the load it withstands. Therefore, as—

$$\text{Load} = \text{Stress} \times \text{Area transmitting load,}$$

$$\text{Resistance} = \text{Stress} \times \text{Area transmitting load.}$$
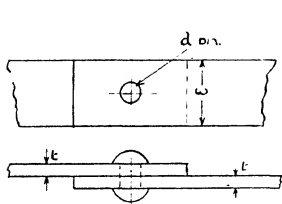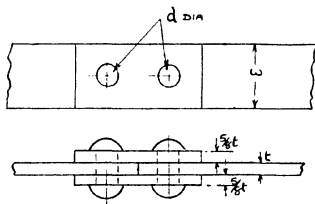
Fig. 54   Fig. 55

Considering the lap joint (Fig. 54)—

Let f_s = Max. Shear Stress in rivet,

and f_t = Max. Tensile Stress in plate.

Max. resistance to tear of plate = Max. resistance to shear of rivet

$f_t \times$ Area transmitting tension $= f_s \times$ Area transmitting shear.

$$f_t\,(w-d)t = f_s\frac{\pi}{4}\,d^2$$

$$\therefore\ w-d = \frac{f_s\pi\,d^2}{f_t\,4\,t}$$

$$w = \frac{f_s\pi\,d^2}{f_t\,4\,t} + d$$

If the plates should be butt-jointed, as in Fig. 55, the rivet will be in double shear, and the equation becomes—

Max. resistance to tear = Max. resistance to shear.

$$f_t\,(w-d)t\ =\ f_s\frac{2\pi\,d^2}{4}$$

$$w\ =\ \frac{f_s\pi\,d^2}{f_t2t}\ +\ d$$

It is found by test that the resistance to shear is not always doubled when the rivet is in double shear, and in consequence this is sometimes taken as

$$f_s\ 1\cdot75\ \frac{\pi}{4}\,d^2.$$

A riveted joint must also resist crushing. If we equate the resistance to crushing to the resistance to shear, we obtain an expression for the diameter in terms of the thickness.

Let f_b = Maximum bearing stress.

Resistance to crushing = Resistance to shear.

For the lap joint (Fig. 54) this gives—

$$f_b dt\ =\ f_s\frac{\pi}{4}\,d^2$$

$$\therefore\ d\ =\frac{f_b 4t}{f_s\pi}$$

$$=\ 1\cdot27\,\frac{f_b t}{f_s}$$

For the butt joint (Fig. 55) it gives—

$$d\ =\ 0\cdot64\frac{f_b t}{f_s}$$

It is usual to take the allowable bearing stress as equal to twice the allowable shear stress of the material—

i.e., $\dfrac{f_b}{f_s}\ =\ 2$

This gives for the lap joint, assuming the plate and rivet are made of the same material—

$$d\ =\ 2\cdot54\ t$$

and for the butt joint

$$d\ =\ 1\cdot27\ t.$$

These values give impossibly large rivets for thick plates, such as are used in boiler work, and it is usual in that case to use the empirical formula—

$$d\ =\ 1\cdot2\ \sqrt{t.}$$

For small plates, however, this empirical formula gives a rivet bigger than $2\cdot54\ t$, with the result that if the allowable shear stress is reached, the allowable bearing stress will be exceeded.

Unfortunately, in aeroplane work, when plates of about 0·015 inch. thick are used, it is quite impracticable to use the small rivets given by the theoretical values. It follows in this case that the strength of the joint will depend on its maximum resistance to crushing. For very thin plates the bearing stress may be lowered in relation to the shear stress by using tubular rivets.

Pitch of Rivets.

A wide plate is joined by one or more rows of rivets, regularly spaced.

The distance between centres of adjacent rivets in any one row is called the pitch.

Thus in the single riveted lap joint (Fig. 56) the distance p represents the pitch.
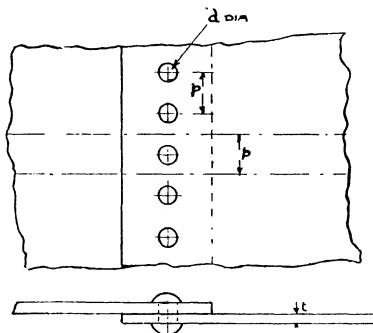
FIG. 56

The plate will be weakened along the row of rivets due to the rivet holes in it.   In order that this loss in strength may be as small as possible, it must be arranged that the pitch is neither too small nor too large.   If the pitch is small there will be more rivets than necessary, leading to an extra weakening of the plate, due to the number of rivet holes.   If the pitch is too large the plate will be stronger, but the joint would be weakened, due to there being less rivets to resist the shear force.   It must therefore be arranged that the maximum resistance to tear of the plate must equal the resistance to shear of the rivets, as for the narrow plates.

Consider the single riveted lap joint (Fig. 56).   It may be divided up into a number of equal strips, each equal in width to the pitch. We can find the width of these strips in the same way as for the single riveted plate of width w.

i.e., Resistance to tear of plate = Resistance to shear of rivet.

$$f_t\,(p-d)t \;=\; f_s\frac{\pi}{4}\,d^2$$

$$p \;=\; \frac{f_s\pi d^2}{f_t\,4\,t} + d.$$

For the double riveted lap joint, Fig. 57 (a), the strip of width p will contain two rivets, which must both shear before the joint fails by the rivets, whilst the minimum area of plate is still $(p-d)t$ as it will fail on one row of rivets as shown in Fig. 57 (b).

For this joint

Resistance to tear = Resistance to shear.

$$f_t\,(p-d)t \;=\; f_s 2\frac{\pi}{4}\,d^2$$

$$p \;=\; \frac{f_s\pi d^2}{f_t\,2\,t} + d.$$

(a)   FIG. 57
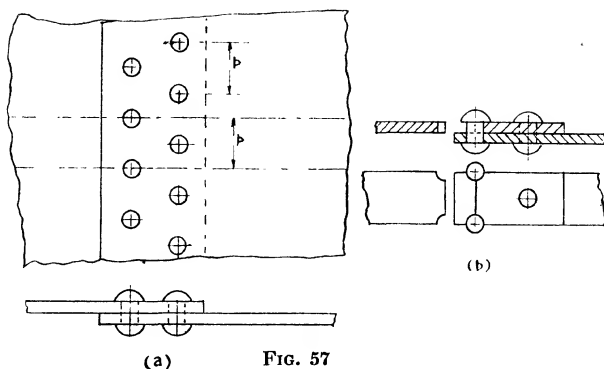
EXAMPLE.—Find the pitch of the rivets on the double riveted butt joint (Fig. 58) if the plate is $\frac{1}{2}$ inch thick, and $\frac{7}{8}$ inch diameter rivets are used.
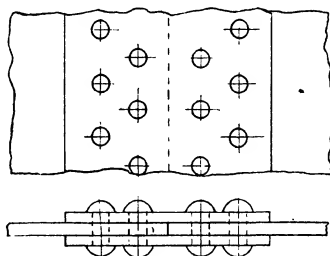
FIG. 58

Take the allowable shear stress of the rivets to equal 0·8 the allowable tensile stress of the plate.

The rivets are in double shear.

Resistance to tear $=$ Resistance to shear.

$$f_t \, (p-d)t \; = f_s 4 \frac{\pi}{4} d^2$$

$$p \; = \frac{f_s \pi d^2}{f_t t} \; + \; d$$

$$= \frac{0 \cdot 8 \times \pi \times (\frac{7}{8})^2}{\frac{1}{2}} + \tfrac{7}{8}$$

$$= \; 3 \cdot 847 + 0 \cdot 875$$

$$= \; 4 \cdot 722 \text{ inches.}$$

Efficiency of Joints.

It is clear that the rivet holes will make the joint weaker than the solid plate.

The ratio of the strength of the joint to the strength of the solid plate is called the efficiency of the joint.

If the resistance to tearing is equal to, or less than, the resistance to shear and crushing,

$$\text{Efficiency} = \frac{\text{Resistance to tear of joint}}{\text{Resistance to tear of solid plate}}$$

$$= \frac{f_t\,(p-d)t}{f_t\,p\,t} = \frac{p-d}{p}$$

If, however, the resistance to shear is less than the resistances to tear and crushing, the efficiency becomes

$$\frac{\text{Resistance to shear}}{f_t p\,t} = \frac{f_s A}{f_t\,p\,t},$$

where A is the area transmitting shear on a strip equal in width to the pitch.

Again, if the resistance to crushing is least,

$$\text{Efficiency} = \frac{\text{Resistance to crushing}}{f_t\,p\,t}$$

$$= \frac{f_b d\,t\,n}{f_t\,p\,t} = \frac{f_b\,d\,n}{f_t\,p}$$

where n is the number of rivets in a strip equal in width to the pitch.

EXAMPLE.—A single riveted butt joint is required for a plate $\frac{1}{2}$ inch thick ; the rivets are to be $\frac{3}{4}$ inch diameter. Find the necessary pitch, and the lowest efficiency, if the maximum allowable stresses are : Tensile stress of plate, 9 tons/sq. inch ; shear stress of rivets, 7 tons/sq. inch : and bearing stress, 14 tons/sq. inch.

Resistance to tear = Resistance to shear.

$$f_t(p-d)t = f_s\,\frac{\pi}{2}\,d^2$$

$$p = \frac{f_s\pi\,d^2}{f_t\,2\,t} + d$$

$$= \frac{7\times22\times2\times3\times3}{9\times7\times2\times4\times4} + 0\text{·}75$$

$$= 1\text{·}375 + 0\text{·}75$$

$$= 2\text{·}125 \text{ or } 2\frac{1}{8} \text{ inches.}$$

As the rivets are larger than $1\text{·}27\,t$, the joint will be weakest in crushing.

$$\text{Crushing efficiency} = \frac{f_b\,d\,n}{f_t\,p}$$

$$= \frac{14\times3\times1}{9\times4\times2\text{·}125}$$

$$= 0\text{·}55 = \text{lowest efficiency.}$$

As the resistance to tear equals resistance to shear, tearing efficiency equals shearing efficiency.

$$= \frac{p-d}{p} = \frac{1 \cdot 375}{2 \cdot 125}$$

$$= 0 \cdot 65.$$

The lowest efficiency 0·55 is the actual efficiency of the joint.

Strength of Rivets in an Aeroplane Spar.

In a built-up aeroplane spar the loads are usually transmitted from the bracing by means of side plates riveted to the spar flanges. An example is shown in Fig. 59.
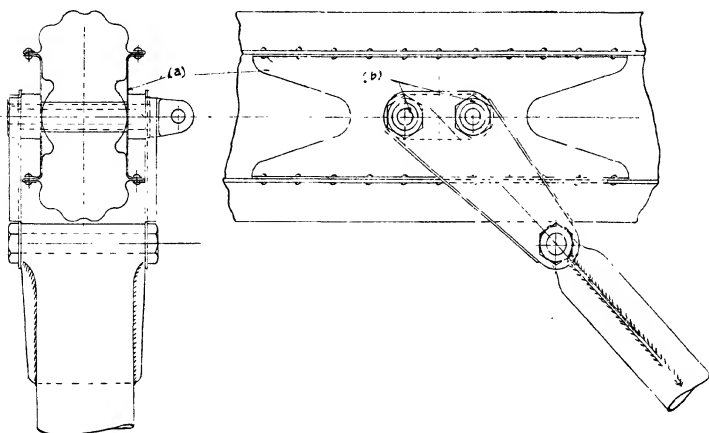
FIG. 59

It is usual to obtain, by means of a stress diagram (see Vol. II), the longitudinal loads in the spar, which will be different on each side of such a fitting. The algebraic difference between these two loads is the force tending to shear the rivets.

The size of the rivets is largely determined by the width of the lip of the spar, and is usually $\frac{3}{32}$ inch diameter.

The load being parallel to the line of rivets, the pitch is not restricted, as in the cases previously considered, but is usually made from $\frac{3}{4}$ inch to 1 inch.

The thickness of the side-plates is always greater than the thickness of the flange, and the web is neglected, as the flange is considered to take all the load. Thus the minimum bearing area is between the rivets and flanges.

The number of rivets required will depend on the resistance to shear and the resistance to crushing. The plates being very wide in comparison to the rivets, there will be no question of them tearing.

Resistance to shear $= f_s \dfrac{\pi}{4} d^2 n.$

Resistance to crushing $= f_b \, d \, t \, n,$

where n is the number of rivets.

EXAMPLE.—The longitudinal load in a spar is increased at a bracing attachment fitting from 4,000 lb. tension to 12,000 lb. tension. The bracing is attached to the spar by means of side plates riveted to the flanges by $\frac{3}{32}$ inch diameter rivets. The allowable shear stress of the rivets is 26 tons/sq. inch, and the allowable bearing stress 52 tons/sq. inch. The thickness of the spar flanges is 0·024 inch. Find the necessary number of rivets.

Load to be transmitted $= 12,000 - 4,000$
$$= 8,000 \text{ lb.}$$

Resistance to shear $= 8000 = f_s \frac{\pi}{4} d^2 n$

$$\therefore \ 8000 = \frac{26 \times 2240 \times 22 \times 3 \times 3 \times n}{4 \times 7 \times 32 \times 32}$$

$$n = \frac{8000 \times 4 \times 7 \times 32 \times 32}{26 \times 2240 \times 22 \times 3 \times 3} = 20$$

Check for crushing—

Resistance to crushing $= f_b d t n$

$$= \frac{52 \times 2240 \times 3 \times 0·024 \times 20}{32}$$

$$= 5,242 \text{ lb.}$$

This is not as great as the 8,000 lb. required, and therefore the joint is weakest in crushing, and the number of rivets must be determined from their resistance to crushing, instead of shear.

Resistance to crushing $= 8000 = f_b d t n$

$$\therefore \ 8000 = \frac{52 \times 2240 \times 3 \times 0·024 \times n}{32}$$

$$n = \frac{8000 \times 32}{52 \times 2240 \times 3 \times 0·024}$$

$$= 31 = \text{number of rivets required.}$$

With thin sheet metal members it will usually be found that the joint is weakest in crushing, except where hollow rivets are used.

EXAMPLE.—Find the number of rivets required in the above example, if the solid rivets are replaced by tubular rivets $\frac{1}{8}$ inch outside diameter and 0·09 inch inside diameter.

Resistance to shear $= 8000 = f_s \frac{\pi}{4} (D^2 - d^2) n$

$$\therefore \ 8000 = \frac{26 \times 2240 \times 22 \times (0·125^2 - 0·09^2) n}{4 \times 7}$$

$$n = \frac{8000 \times 4 \times 7}{26 \times 2240 \times 22 (0·125^2 - 0·09^2)}$$

$$= \frac{8000 \times 4 \times 7}{26 \times 2240 \times 22 \times 0·00753}$$

$$= 24.$$

Check for crushing—

Resistance to crushing $= f_b d\,t\,n$

$$= \frac{52 \times 2240 \times 0{\cdot}024 \times 24}{8}$$

$$= 8{,}387 \text{ lb.}$$

This is greater than the resistance to shear, therefore 24, the number of rivets required to resist shear, is the number required for the joint.

If the diameter/thickness ratio is large, hollow rivets are likely to collapse due to the shell buckling inwards before shear failure occurs. In this case the shear strength should be determined from test.

Torsional Resistance of a Riveted Joint.

It often happens that two members are riveted together in such a manner that the load transmitted does not pass through the centroid of the group of rivets.
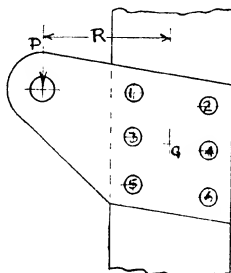
FIG. 60

Consider the case of the bracket (Fig. 60). The rivets have to resist a direct load P and a torque PR, R being the perpendicular distance of the line of action of P from G, the centroid of the rivets cross-sectional areas. The actual loads taken by individual rivets in practice will depend largely on the relative tightness of the rivets in the holes— e.g., if one rivet is loose, it will not take any of its share of the load until the clearance has been taken up, and before this can be done the other rivets must have been strained. This, however, applies to all riveted joints, and as calculations must be based on the ideal case it shows the necessity of good fitting.

In calculating the loads on the rivets, the following assumptions are made—

    (1) Stress is proportional to strain ;

    (2) The plate is not distorted ;

    (3) The fit of the rivets is the same in each case ;

    (4) Young's Modulus is the same for all rivets.

There will be a direct force parallel to P taken by each rivet equal to $\dfrac{Aa}{P} = \dfrac{P}{N}$ where rivets are the same diameter,

where A is the total cross-sectional area,

      a the cross-sectional area of the rivet concerned,

and N the number of rivets

There will also be a force on each rivet acting at right-angles to the line joining the centre of the river to G, such that the combined effect of these forces multiplied by their respective distances from G will equal the torque PR.

The rivets will be slightly strained when subjected to the torque PR, with the result that there will be a minute turning of the plate about G.

Let the plate turn through the angle θ; then, referring to Fig. 61, it will be seen that for rivets 1 and 2 the strain is proportional to $l_1\theta$ and $l_2\theta$ respectively, where l_1, l_2, etc., represent the respective distances of the rivets 1, 2, etc., from G.

i.e., $\dfrac{\text{Strain 1}}{\text{Strain 2}} = \dfrac{l_1\theta}{l_2\theta} = \dfrac{l_1}{l_2}$.

But Stress is proportional to Strain,

and stress $f = \dfrac{F}{a}$

where $F =$ force in each rivet

and $a =$ its cross-sectional area.

$$\therefore \frac{l_1}{l_2} = \frac{f_1}{f_2} = \frac{F_1 a_2}{F_2 a_1}$$

$$\therefore F_1 = \frac{F_2 l_1 a_1}{l_2 a_2} = \text{Force in rivet (1).}$$
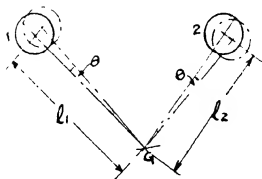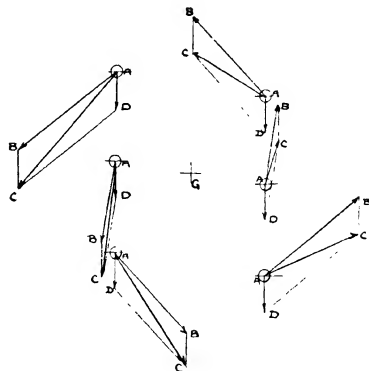
FIG. 61   FIG 62

Its moment about G

$$= F_1 l_1 = \frac{F_2 l_1^2 a_1}{l_2 a_2}.$$

$PR =$ Sum of moments of force in all rivets about G

$$= F_1 l_1 + F_2 l_2 + F_3 l_3 + \text{etc.}$$

$$= \Sigma Fl$$

$$= \frac{F_2 \Sigma l^2 a}{l_2 a_2}$$

$$\therefore F_2 = \frac{PR\, l_2 a_2}{\Sigma l^2 a}$$

If the cross-sectional areas of the rivets are the same, this equation becomes

$$F_2 = \frac{PRl_2}{\Sigma l^2}$$

i.e., the force in any rivet due to the torque is equal to the product of the torque and distance of the rivet from G divided by the sum of the squares of the distances from G of all the rivets.

The total force exerted by any rivet may now be found by adding geometrically the force due to direct load and the force due to torque. This may be done graphically by drawing the parallelogram of forces for each rivet.   Referring to Fig. 62, for each case—

> AB represents the force due to torque,
>
> AD represents the direct force,
>
> AC represents the total resultant force.

EXAMPLE 1.—Find the size of rivets required for the bracket shown in Fig. 63.   The plate is 0·064 inch thick and the rivets, which are to be all the same size, are in single shear.

Take 12 tons/sq. inch as the permissible shear stress.

Direct load in each rivet $= \dfrac{800}{5} = $ 160 lb.

Distance of line of action of 800 lb. load from centroid

$$= 2 \cos 30° = 1·732 \text{ inch.}$$

Distance of rivets 1, 2, 4, 5 from centroid $= \sqrt{2} = 1·414$ inch.

Distance of rivet 3 from centroid $= 0.$

Load due to torque in rivets 1, 2, 4, 5 $= \dfrac{800 \times 1·732 \times 1·414}{4 \times 2}$

$$= 245 \text{ lb.}$$

Load due to torque in rivet 3 $= 0$ lb.

The resultant force on each rivet is found graphically in Fig. 64. This shows the maximum force on any rivet to be in rivet (1), and equal to 400 lb., from which the diameter of the rivets may be obtained in the usual manner.

$$\text{Thus } f_s = \frac{\text{Load}}{\text{Area}}$$

$$12 \times 2240 = \frac{400}{\frac{\pi}{4} d^2}$$

$$d = \sqrt{\frac{400 \times 4}{\pi \times 12 \times 2240}}$$

$$= \sqrt{0·019}$$

$$= 0·138 \text{ inch.}$$

Say $\frac{5}{32}$ inch diameter.

Check for crushing—

$$f_b = \frac{\text{Load}}{dt}$$

$$= \frac{400 \times 32}{5 \times 0.064}$$

$$= 40,000 \text{ lb./sq. inch}$$

$$= 18 \text{ tons/sq. inch.}$$

This is well under $2f_s$, therefore the rivets will withstand crushing.
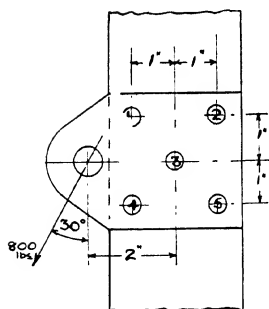
FIG. 63
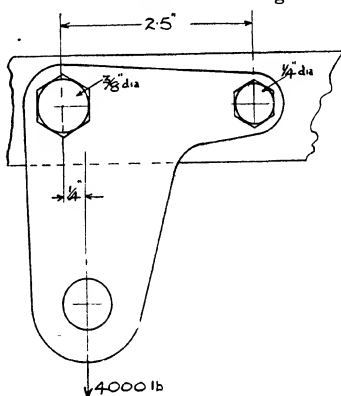
FIG. 65
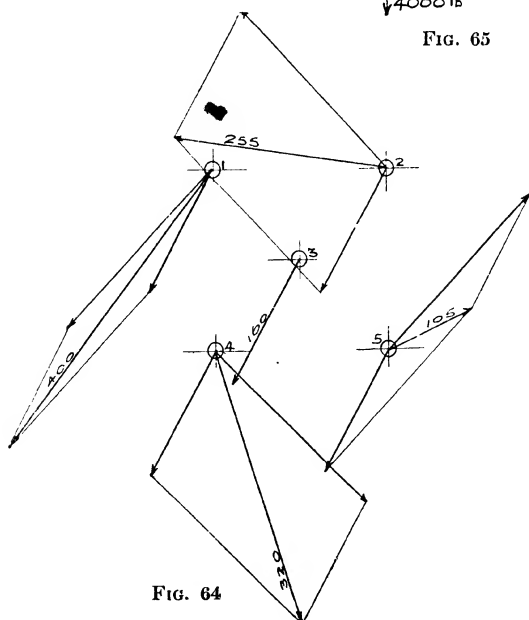
FIG. 64

EXAMPLE 2.—Find the shear stress in the two attachment bolts on the fitting in Fig. 65, if they are in single shear, and the pull of 4,000 lb. is at right-angles to the line joining the bolt centres.

Area of $\frac{3}{8}$ inch bolt $= \frac{\pi}{4} \times 0 \cdot 375^2 = 0 \cdot 11$ sq. inch.

Area of $\frac{1}{4}$ inch bolt $= \frac{\pi}{4} \times 0 \cdot 25^2 = 0 \cdot 049$ sq. inch.

Direct load on $\frac{3}{8}$ inch bolt $= \dfrac{Pa}{A} = \dfrac{4000 \times 0 \cdot 11}{0 \cdot 11 + 0 \cdot 049}$

$$= \frac{440}{0 \cdot 159} = 2,767 \text{ lb.}$$

Direct load on $\frac{1}{4}$ inch bolt $= 4000 - 2767 = 1,233$ lb.

Let the distance of centroid of bolt areas from centre of $\frac{3}{8}$ inch bolt $= l_1$.

Then $l_1 (0 \cdot 11 + 0 \cdot 049) = 2 \cdot 5 \times 0 \cdot 049$.

$$0 \cdot 159 \, l_1 = 0 \cdot 1225$$

$$l_1 = \frac{0 \cdot 1225}{0 \cdot 159}$$

$$= 0 \cdot 77 \text{ inch.}$$

Distance of centroid from centre of $\frac{1}{4}$ inch bolt

$$= l_2 = 2 \cdot 5 - 0 \cdot 77 = 1 \cdot 73 \text{ inch.}$$

Torque $= PR = 4000 (0 \cdot 77 - 0 \cdot 25)$

$$= 4000 \times 0 \cdot 52$$

$$= 2080 \text{ inch lb.}$$

Load due to torque in $\frac{3}{8}$ inch bolt

$$= \frac{PR \times l_1 a_1}{\Sigma l^2 a}$$

$$= \frac{2080 \times 0 \cdot 77 \times 0 \cdot 11}{0 \cdot 77^2 \times 0 \cdot 11 + 1 \cdot 73^2 \times 0 \cdot 049}$$

$$= \frac{2080 \times 0 \cdot 77 \times 0 \cdot 11}{0 \cdot 2119}$$

$$= 832 \text{ lb.}$$

Load due to torque in $\frac{1}{4}$ inch diameter bolt

$$= \frac{2080 \times 1 \cdot 73 \times 0 \cdot 049}{0 \cdot 2119}$$

$$= 832 \text{ lb.}$$

Total load in $\frac{3}{8}$ inch bolt $= 2767 + 832$

$$= 3,599 \text{ lb.}$$

Total load in $\frac{1}{4}$ inch bolt $= 1233 - 832$

$$= 401 \text{ lb.}$$

Shear stress in $\frac{3}{8}$ inch bolt $= \dfrac{3599}{0\cdot 11}$

$= 32{,}720$ lb./sq. inch.

Shear stress in $\frac{1}{4}$ inch bolt $= \dfrac{401}{0\cdot 049}$

$= 8{,}180$ lb./sq. inch.

Tie Bars and Wiring Lugs.

If a tie bar is bolted or riveted to its end fitting, it has been shown that the bar is weakened at the bolt or rivet hole.   It follows that, if the bar is of uniform cross-section, it is over strength at all parts except the joints.

In the structure of the aeroplane, where the reduction of weight is of vital importance, it would be necessary to reduce the cross-section between the joints, and so make the bar of uniform strength throughout. Fig. 66 shows how such a reduction may be carried out.
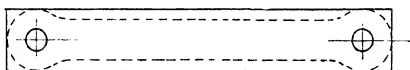
FIG. 66

In light structures and in the aeroplane it is usual to have wires for tension bracing.   These are generally attached at their ends to a strip of sheet metal, called a wiring lug, by means of a fork-end and pin, as illustrated in Fig. 67.
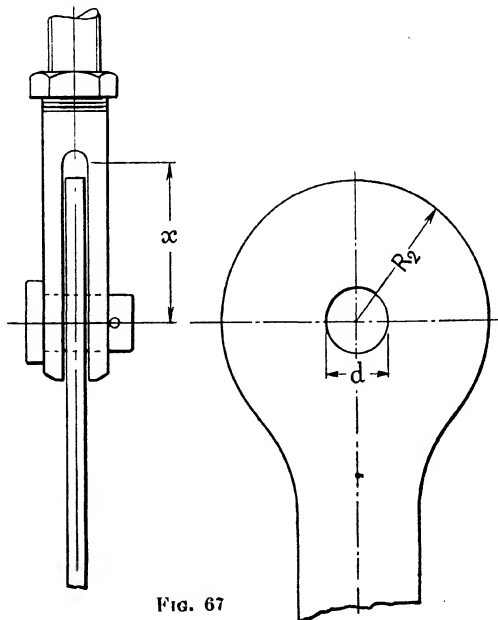
FIG. 67

The minimum diameter of wire (D) may be obtained as follows—

$$f_t = \frac{P}{A} = \frac{P}{\frac{\pi}{4}D^2}$$

$$\therefore D = \sqrt{\frac{4P}{\pi f_t}}$$

where P = maximum load transmitted.

The pin is in double shear, and its diameter

$$d = \sqrt{\frac{2P}{\pi f_s}}$$

In practice the size of wire will determine the fork-end to be used, and as these are standardised the size of pin and pin-hole is fixed.

The thickness of the lug must be such that

$$dt = \frac{P}{f_b}$$

where d = diameter of pin-hole in lug,

t = thickness of lug.

This thickness must also be such that R_2 is less than x (Fig. 67), so as to leave a clearance between the lug and the fork-end.

It should be obvious that the thicker the lug, the smaller may R_2 be made.

There is some doubt about the distribution of stress in a lug, and the proportions are usually from formulæ derived from tests. The " offset " lug shown in Fig. 68 is of practically uniform strength ; many designers, however, use the concentric one, shown dotted.
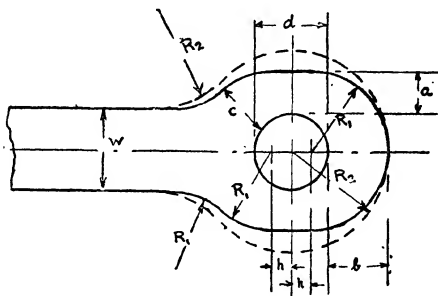
FIG. 68

Suitable formulæ for the design of lugs of either type are given below—

$$W = \frac{P}{tf_t}$$

$$a = 0{\cdot}555\ W.$$

$$R_1 = 0{\cdot}555\ W + \frac{d}{2}.$$

$$R_2 = 0{\cdot}695\ W + \frac{d}{2}.$$

$$b = 1{\cdot}25\ a = 0{\cdot}695\ W.$$

$$h = R_2 - R_1 = 0{\cdot}14\ W.$$

The reason why c is greater than a and why a is greater than $\dfrac{W}{2}$ may be understood by considering an exaggerated lug (Fig. 69). By drawing the triangle of forces for the forces at the point x, it will be seen that the forces AB and BC are greater than half the force AC, due to their angularity. Again, considering the triangle of forces for the point y, there must be a force FD exerted by the material resisting the tendency for the lug to bend inwards at this point. The lug must therefore withstand bending as well as direct load at y.
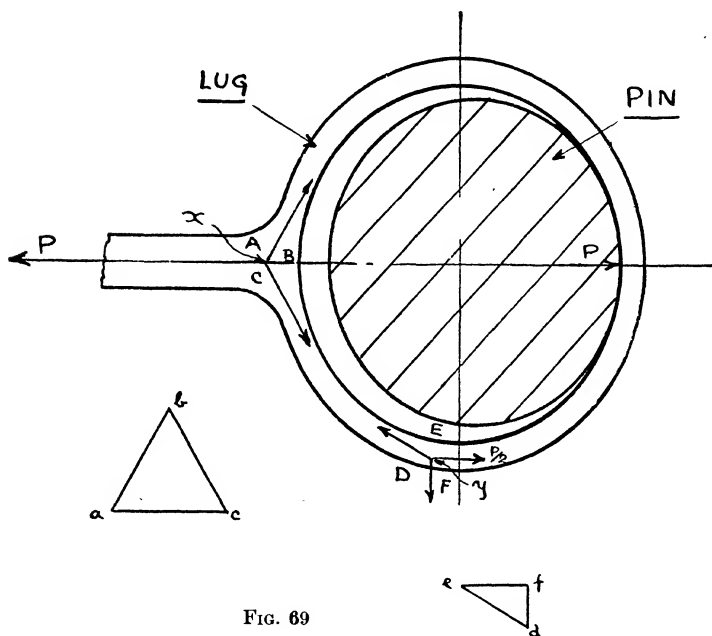
Fig. 69

EXAMPLE.—Design a wiring lug to carry a load of 3,450 lb. The maximum tensile stress is not to exceed 20 tons/sq. inch. The fork end is 0·2 inch wide and 0·41 inch deep from pin centre to bottom of fork. A ¼ inch diameter pin is used.

Take bearing stress to equal twice tensile stress.

$$f_b = \frac{P}{dt}$$

$$\therefore t = \frac{P}{f_b d}$$

$$= \frac{3450 \times 4}{40 \times 2240 \times 1}$$

$$= 0·154 \text{ inch.}$$

Say 8 S.W.G. $= 0·16$ inch thick.

This gives a clearance in the jaw of the fork, but not enough to warrant a bush.

$$W = \frac{P}{tf_t} = \frac{3450}{0\cdot16 \times 20 \times 2240}$$
$$= 0\cdot48 \text{ inch.}$$

Next find R_2, for if this is greater than 0·41, a thicker lug must be used, so that it may be reduced to clear the fork-end.

$$R_2 = 0\cdot695\ W + \frac{d}{2}$$
$$= 0\cdot695 \times 0\cdot48 + 0\cdot125$$
$$= 0\cdot334 + 0.125$$
$$= 0\cdot459 \text{ inch.}$$

This is too large to clear the fork. The thickness must be increased until 0·695 W is less than 0·41 − 0·125 = 0·285 ; *i.e.*, thickness must be greater than $\dfrac{0\cdot16 \times 334}{285}$

$$= 0\cdot188 \text{ inch.}$$

Say, 6 S.W.G. $= 0\cdot192$ inch thick.

Note.—This still gives the required clearance across the jaw.

$$\text{Now } W = \frac{3450}{0\cdot192 \times 20 \times 2240}$$
$$= 0\cdot4 \text{ inch.}$$

$$R_2 = 0\cdot695 \times 0\cdot4 + 0\cdot125$$
$$= 0\cdot278 + 0\cdot125$$
$$= 0\cdot403 \text{ inch.}$$

$$R_1 = 0\cdot555\ W + \frac{d}{2}$$
$$= 0\cdot555 \times 0\cdot4 + 0\cdot125$$
$$= 0\cdot222 + 0\cdot125$$
$$= 0\cdot347 \text{ inch.}$$

$$h = R_2 - R_1$$
$$= 0\cdot403 - 0\cdot347$$
$$= 0\cdot056 \text{ inch.}$$

The lug is given in Fig. 70 for both the concentric and offset cases.
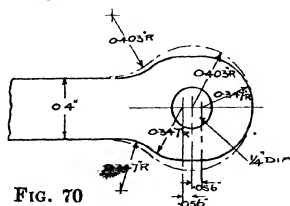
Fig. 70

Thin Cylinders with Internal Pressure.

When a thin cylinder is subjected to internal pressure, this pressure acting normal to the walls will produce a tensile force in the material tangential to the circumference, which will tend to burst the cylinder along a longitudinal section. This is generally called Hoop Tension.

Where the ends are not supported externally against the tendency of the pressure to force them apart, but are held together by the material of the cylinder, there will be a tensile force in the material parallel to the longitudinal axis, usually called longitudinal tension. This force will tend to burst the cylinder across a circumferential cross-section.

Hoop Stress.—Consider the length of cylinder (Fig. 71) and let—

$$d = \text{internal diameter,}$$
$$t = \text{thickness of walls,}$$
$$p = \text{internal gauge pressure,}$$
$$l = \text{length considered.}$$
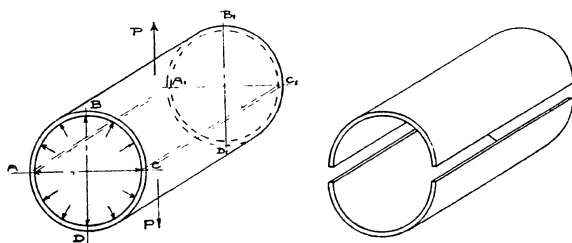
FIG. 71

The half ABC above the plane AC and the half ADC below the plane AC tend to be forced apart along the plane AC by the resultant force P of the pressure on each half acting at right-angles to AC. This tendency is resisted by the strips of material AA_1 and CC_1.

The resultant force P will not equal the product of the pressure, length and half circumference, as only at BB_1 on the curved surface is the pressure acting in the direction of P.
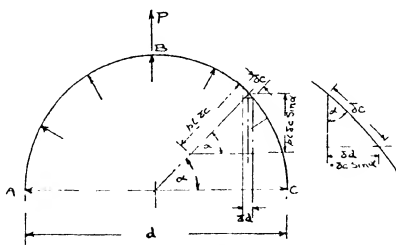
FIG. 72

The force on any element of arc δc (Fig. 72) is equal to $pl\delta c$, and its component in the direction of P equals $pl\delta c \sin \alpha$.

But $\delta c \sin \alpha = \text{projection of } \delta c \text{ on } AC = \delta d$, .

therefore $pl\delta c \sin \alpha = pl\delta d$.

The total force P will equal the sum of the component forces on all the elements $l\delta c$ along the arc ABC

$$= P = pl\Sigma\delta d = pld.$$
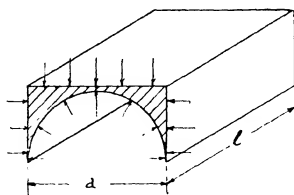
FIG. 73

It will be seen that this force P is the same as though the pressure p were acting on the rectangular area ACC_1A_1. This result is obvious if we consider the block, Fig. 73. The pressure of the air p_1 is acting all over it normal to the surface. The total downward force on the flat top, due to atmospheric pressure, is equal to p_1ld, and as the block does not tend to rise in the air, the total upward force on the curved undersurface must be also equal to p_1ld.

$$\text{Hoop Stress} = \frac{P}{\text{Area transmitting tension}}$$

$$= \frac{pld}{2tl}$$

$$= \frac{pd}{2t}$$

Longitudinal Stress.—The longitudinal tension P_1 will be equal to the product of the pressure, and the cross-sectional area of the cylinder at the section considered, independent of the shape and size of the ends.

$$\text{Thus Longitudinal Stress} = \frac{P_1}{\text{Area transmitting tension}}$$

$$= \frac{p\frac{\pi}{4}d^2}{\pi dt}$$

$$= \frac{pd}{4t}$$

It will be noticed that the hoop stress is double the longitudinal stress, where there are no joints. In a cylinder such as a boiler, it follows that the circumferential riveted joints need only be half as strong as the longitudinal joints—*i.e.*, it may be a less efficient joint.

EXAMPLE.—A steam pipe is 4 inches diameter and $\frac{1}{8}$ inch thick. Find the maximum stress in the pipe when the steam pressure is 120 lb./sq. inch.

The maximum stress will be Hoop Stress.

$$= \frac{pd}{2t}$$

$$= \frac{120 \times 4 \times 8}{2 \times 1}$$

$$\text{Hoop Stress} = 1,920 \text{ lb./sq. inch.}$$

EXAMPLE.—A 12-inch diameter seamless steel tube is closed at the ends with separate end pieces, riveted to the tube by a single row of 50 rivets, as shown in Fig. 74. The vessel thus formed has to withstand an internal pressure of 120 lb./sq. inch.
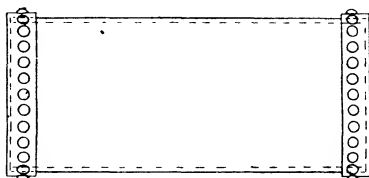
. FIG. 74

Find the diameter of the rivets and the thickness of the tube. The ultimate shear stress of the rivets is 24 tons/sq. inch, and the ultimate tensile stress of the tube 30 tons/sq. inch. Allow a factor of safety of 8.

Let D = diameter of tube,

d = diameter of rivets,

F = Factor of safety.

Longitudinal tension $P_1 = p\,\dfrac{\pi}{4}D^2$

Shear stress of rivets $f_s = \dfrac{P_1 F}{\dfrac{\pi}{4}\,d^2\,n}$

$$\therefore\ d = \sqrt{\dfrac{4P_1}{f_s \pi\,n}}$$

$$= \sqrt{\dfrac{4\,p\pi\,D^2\,F}{f_s \pi\,n\,4}}$$

$$= \sqrt{\dfrac{4\times 120\times \pi \times 144 \times 8}{24\times 2240 \times \pi \times 50 \times 4}}$$

$$= \sqrt{0{\cdot}05143}$$

$$= 0{\cdot}227\ \text{inch.}$$

Say, $\tfrac{1}{4}$ inch diameter.

Tearing efficiency of joint $= \dfrac{\text{Circumference of tube} - d\,n}{\text{Circumference of tube}}$

$$= \dfrac{12\pi - 0{\cdot}25 \times 50}{12\pi}$$

$$= 67\ \text{per cent.}$$

As the tube has not been weakened 50 per cent. circumferentially by the rivet holes, it will tend to fail longitudinally by Hoop tension. The necessary thickness will be that required to withstand Hoop tension.

Hoop stress $f_t = \dfrac{pDF}{2t}$

$$\therefore\ t\ =\ \frac{pDF}{2f_t}$$

$$=\ \frac{120 \times 12 \times 8}{2 \times 30 \times 2240}$$

$$=\ 0.086 \text{ inch thick.}$$

Checking for crushing—

$$2.54\ t\ =\ 2.54 \times 0.086$$

$$=\ 0.22 \text{ inch.}$$

As this is smaller than the rivet diameter, the rivet will fail by shear before crushing.

CHAPTER VIII

SPECIAL CASES

Effect of Bolt Threads.

In designing a fitting care must be taken to ensure that the threaded portion of bolts is not relied on to resist crushing, as the bearing area at the thread is negligible.

There is a small radius under the bolt head, so that if there is no washer between the member and the bolt head, the bolt hole must be slightly chamfered, to allow the bolt to fit home. This chamfered portion must not be considered as providing any bearing area. In dealing with thin plates, it is absolutely necessary to fit a washer, otherwise there would only be line contact, as shown in Fig. 75 (*a*). Incorrect and correct methods of fitting bolts are given in Fig. 75 (*b*) and (*c*) respectively.

(a)

(b)
INCORRECT

(c)
CORRECT
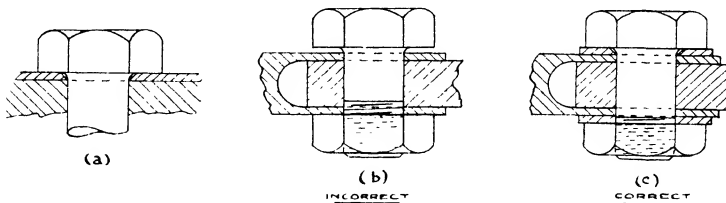
FIG. 75

When a bolt is subjected to tension, as in Fig. 76, the tensile stress must be calculated on the cross-sectional area at the bottom of the thread, as it is here the stress will be at a maximum. The threads will also be tending to shear in the nut, but with standard Whitworth or B.S.F. threads sufficient resistance is obtained if the height of the nut is equal to the diameter of the bolt.
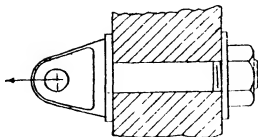
FIG. 76

Where a nut has to be tightened hard up on a bolt, the tensile stress will be considerably increased, and a larger bolt than is required for the applied load must be used.

Bolts of Uniform Strength.

A tension bolt may be made lighter by reducing the cross-sectional area of the unthreaded portion. This may be done by reducing the diameter, but a portion of the full diameter should be left at each end

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to fit the hole ; this is shown in Fig. 77 (*a*).   Another method is to drill a hole from the bolt head to near where the threads start, as shown in Fig. 77 (*b*).
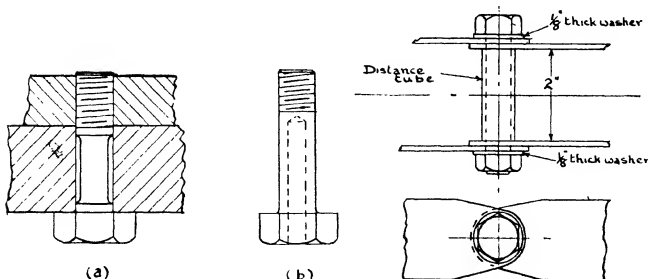
Fig. 77   Fig. 78

It is usual to make the reduced portion of slightly less cross-sectional area than at the bottom of the thread, thus ensuring that the maximum stress does not come on the threaded portion, where a fatigue crack is more likely to occur, if the stress fluctuates.

When bolts are subjected to shear in a thin metal fitting, such as the eyebolts *b* in Fig. 59, a large diameter is required to provide sufficient bearing area.   If the bolts are made solid they will be much heavier than required, as a much smaller cross-sectional area than that given by the full diameter is sufficient to resist shear.   It is usual in these cases to use a hollow bolt.   As there is no load on the threads, the hole may extend right through the threaded portion, provided, of course, the diameter of the hole is appreciably smaller than the diameter at the bottom of the threads.   It may be seen that with the eye-bolts in Fig. 59 the hole must be drilled from the threaded end.

EXAMPLE 1.—Design a bolt of minimum weight for the fitting shown in Fig. 78.

The plates are 0·128 inch thick, and the load to be transmitted is 15,300 lb.   Take the bearing and shear stresses to be 120,000 lb./sq. inch and 54,000 lb./sq. inch respectively.

$$\text{Let } D = \text{outside diameter}$$

$$\text{and } d = \text{inside diameter.}$$

$$\text{Load on each side } = \frac{15300}{2} = 7650 \text{ lb.}$$

$$\text{Load } = P = f_b D t.$$

$$D = \frac{P}{f_b t}$$

$$= \frac{7650}{120000 \times 0\cdot128}$$

$$= 0\cdot5 \text{ inch.}$$

$\frac{1}{2}$ inch outside diameter bolt.

$$P = f_8 \frac{\pi}{4}(D^2 - d^2)$$

$$\therefore D^2 - d^2 = \frac{P4}{f_8 \pi}$$

$$\therefore d = \sqrt{D^2 - \frac{P4}{f_8 \pi}}$$

$$= \sqrt{0.5^2 - \frac{7650 \times 4}{54000 \times \pi}}$$

$$= \sqrt{0.25 - 0.1805}$$

$$= \sqrt{0.0695}$$

$$= 0.2636 \text{ inch.}$$

Say, $\frac{1}{4}$ inch inside diameter.

EXAMPLE 2.—If the plates in Example 1 were 0·064 inch thick, what would be the dimensions of the bolt ?

$$D = \frac{7650}{120000 \times 0.064}$$

$$= 1 \text{ inch outside diameter.}$$

$$d = \sqrt{1 - \frac{7650 \times 4}{54000 \times \pi}}$$

$$= \sqrt{1 - 0.1805}$$

$$= \sqrt{0.8195}$$

$$= 0.905 \text{ inch inside diameter.}$$

This inside diameter gives too thin a wall to the bolt, also the diameter at the bottom of the threads is smaller. Using a B.S.F. thread, the maximum inside diameter would be $\frac{3}{4}$ inch, to allow the hole to pass through the threaded portion.
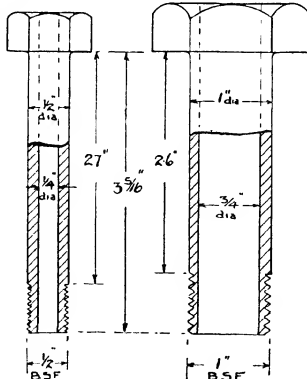
FIG. 79

When the plates are very thin, as in the second example, the crushing resistance will determine the outside diameter, but the inside diameter will have to be made small enough to give a reasonable thickness to the walls, which will be a smaller diameter than that determined by the resistance to shear. The bolts for these two examples are given in Fig. 79.

Tubes in Tension.

It has already been shown, in Chapter VII, that a tension member is weakened at the joint. This fact must be remembered when dealing with tubes which have to resist tension. It is usual to attach a tube to a fitting by means of a socket. This socket is pinned to the tube by taper pins on ferrules, as illustrated in Fig. 80 (a) and (b) respectively.

(a)   (b)
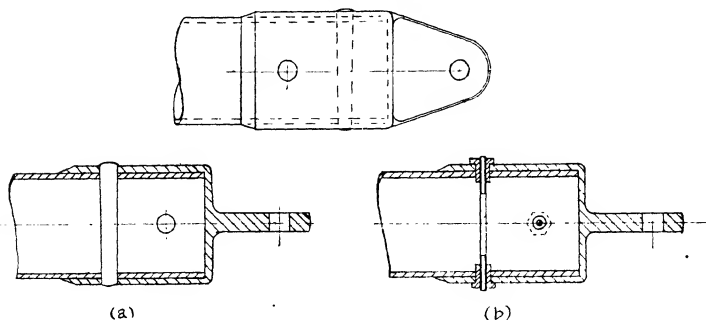
FIG. 80

The pin holes will reduce the effective cross-sectional area of the tube from $\pi D t$ to $\pi D t - 2 d t$

$$= t\,(\pi D - 2d),$$

where D = diameter of tube,

d = diameter of pin or ferrule,

t = thickness of tube.

EXAMPLE 1.—A steel tube which is 1 inch diameter and 0·025 inch thick is fitted at each end with a forked socket, by means of two $\frac{3}{16}$ inch diameter taper pins. Find the maximum tensile and bearing stresses in the tube when it is subjected to a tension of 3,000 lb.

$$\text{Bearing stress} = \frac{3000}{4dt}$$

$$= \frac{3000 \times 16}{4 \times 3 \times 0{\cdot}025}$$

$$= 160{,}000 \text{ lb./sq. inch.}$$

$$\text{Tensile stress} = \frac{3000}{t\,(\pi D - 2d)}$$

$$= \frac{3000}{0{\cdot}025\,(\pi \times 1 - 2 \times \frac{3}{16})}$$

$$= \frac{3000}{0{\cdot}025 \times 2{\cdot}767}$$

$$= 43{,}400 \text{ lb./sq. inch.}$$

* EXAMPLE 2.—A tensile load of 7,200 lb. has to be transmitted by a tube. The attachment is made by sockets, pinned to the tube by three taper pins. Find the diameter of tube and taper pins, if a 20 S.W.G. (0·036 inch thick) tube is used. Take—

Allowable tensile stress in tube   40 tons/sq. inch.

Allowable shear stress in pins   20 tons/sq. inch.

Allowable bearing stress   40 tons/sq. inch.

$$\text{Bearing stress } f_b = \frac{P}{6\,dt}$$

$$\therefore\ d = \frac{P}{f_b\,6\,t}$$

$$= \frac{7200}{40 \times 2240 \times 6 \times 0·036}$$

$$= 0·37 \text{ inch.}$$

Say ⅜ inch diameter pins.

Check for shear—

$$f_s = \frac{P}{6\frac{\pi}{4}\,d^2}$$

$$= \frac{7200 \times 8 \times 8 \times 4}{6 \times \pi \times 3 \times 3 \times 2240}$$

$$= 4·85 \text{ tons/sq. inch.}$$

$$\text{Tensile stress } f_t = \frac{P}{t\,(\pi D - 2d)}$$

$$\therefore\ D = \frac{\dfrac{P}{f_t\,t} + 2d}{\pi}$$

$$= \frac{\dfrac{7200}{40 \times 2240 \times 0·036} + \dfrac{2 \times 3}{8}}{\pi}$$

$$= \frac{2·23 + 0·75}{3·14}$$

$$= 0·95 \text{ inch.}$$

1 inch diameter tube.

Loads in Duplicate Supports.

If a load has more than one support, such as the weight supported by two wires A and B in Fig. 81, the load will be divided equally between the two only when they are the same length, cross-sectional area, and have the same Young's Modulus.

As it is practically impossible to make the wires exactly the same length, it is usual to assume that each wire takes more than half the load.

4—(A.9002)

If wire A is slightly longer than wire B, it will not take any load until wire B has stretched an amount equal to the difference in length, after which they will both stretch together.
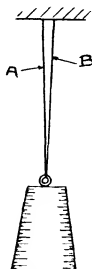
FIG. 81

Let P_A and P_B = loads in wires A and B respectively.

L_A and L_B = length of wires A and B respectively.

A_A and A_B = cross-sectional areas of wires A and B respectively.

e_A and e_B = elongation of wires A and B respectively.

E_A and E_B = Young's Modulus of wires A and B respectively.

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{PL}{Ae}$$

$$\therefore e = \frac{PL}{AE}$$

$$e_B = e_A + \text{difference in length.}$$

$$\therefore \frac{P_B L_B}{A_B E_B} = \frac{P_A L_A}{A_A E_A} + \text{difference in length.}$$

also $P_A + P_B$ = Total load.

From these two equations the load in each wire may be found.

EXAMPLE 1.—A total load of 8,000 lb. has to be transmitted by duplicated lift wires. Due to faulty rigging, one wire has a length of 12 ft. 6·5 in. and the other 12 ft. 6·3 in., when not subjected to load. If the cross-sectional area of each wire is 0·05 sq. in., find the load taken by each wire. $E = 30,000,000$ lb. sq. inch.

Let A stand for long wire, and B for short wire.

Then $P_A + P_B$ = 8000 lb.

$$P_A = 8000 - P_B.$$

$$e_B = e_A + 0·2.$$

$$\frac{P_B L_B}{A E} = \frac{P_A L_A}{A E} + 0·2.$$

$$\therefore P_B L_B = P_A L_A + 0·2 \, AE.$$

Substituting for P_A,

$$P_B L_B = (8000 - P_B) L_A + 0.2\ AE$$

$$P_B(L_A + L_b) = 8000 L_A + 0.2\ AE$$

$$P_B = \frac{8000\ L_A + 0.2\ AE}{L_A + L_B}$$

$$= \frac{8000 \times 150.5 + 0.2 \times 0.05 \times 30000000}{300.8}$$

$$= 5,000 \text{ lb. load in short wire.}$$

Load in long wire $= 8000 - 5000$

$$= 3,000 \text{ lb.}$$

EXAMPLE 2.—A symmetrical block weighing 30 tons rests on three cylindrical steel supports. The centre one is exactly under the C.G. of the block, and the other two are an equal distance each side of it in a straight line. The centre cylinder is 3-inch diameter and the other two 2-inch diameter ; they are all 3 inches in height. When lowering the block into position it was found that when the block was just touching the outer cylinders there was a clearance of 0.0008 inch on the centre cylinder. Neglecting any distortion of the block, find the load taken by each cylinder.

Take $E = 13,000$ tons/sq. inch.

Let A stand for outer supports and B for centre support.

$$A_A = \frac{\pi}{4} 2^2 = 3.14 \text{ sq. inches.}$$

$$A_B = \frac{\pi}{4} 3^2 = 7.07 \text{ sq. inches.}$$

$$2\ P_A + P_B = 30$$

$$P_B = 30 - 2\ P_A.$$

$$e_A = e_B + 0.0008.$$

$$\frac{P_A L}{A_A E} = \frac{P_B L}{A_B E} + 0.0008.$$

$$\frac{P_A}{A_A} = \frac{P_B}{A_B} + \frac{0.0008\ E}{L}.$$

$$\frac{P_A}{A_A} = \frac{30 - 2\ P_A}{A_B} + \frac{0.0008\ E}{L}$$

$$\therefore\ P_A\left(\frac{1}{A_A} + \frac{2}{A_B}\right) = \frac{30}{A_B} + \frac{0.0008\ E}{L}$$

$$P_A\left(\frac{1}{3.14} + \frac{2}{7.07}\right) = \frac{30}{7.07} + \frac{0.0008 \times 13000}{3}.$$

$$P_A \times 0.601 = 4.242 + 3.467.$$

$$P_A = \frac{7.709}{0.601}$$

$$= 12.83 \text{ tons.}$$

$$P_{\text{B}} = 30 - 2 \times 12\cdot83$$
$$= 30 - 25\cdot66$$
$$= 4\cdot34 \text{ tons.}$$

i.e., Load in centre support = 4·34 tons.

Load in outer supports = 12·83 tons each.

Suddenly Applied Loads.

Consider the rod (Fig. 82) of length *l*. A weight *W* falls freely from a height *h* on to the projection at the lower end of the rod.
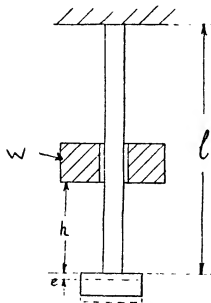
FIG. 82

Let the distance the rod elongates = *e*,

then the total distance *W* has fallen = *h+e*.

The work done by the falling weight will equal the potential energy it has lost = *W* (*h* + *e*).

This also equals the work done in stretching the rod.

The work done in stretching a tensile member will equal the average load multiplied by the extension. If the member is not stressed past the limit of proportionality, the average load will equal half the maximum load.

Thus work done $= \dfrac{\text{Maximum load}}{2} \times \text{Extension.}$

We have—

Work done by falling weight = Work done in stretching rod.

$$W(h+e) = \frac{\text{Max. load in rod}}{2} \times e$$

$$\therefore \text{ Max. load in rod} = 2\,W\!\left(\frac{h}{e}+1\right)$$

When *h* = o,

Max. load in rod = 2 *W*.

It follows that if a load is applied to a member suddenly, instead of by gradually increasing it from zero, the actual force the member has to withstand is double the load applied.

EXAMPLE.—A load of 400 lb. is dropped 2 inches on to a collar on the end of a steel rod 12 feet long and 1 inch diameter. Find the stress produced.

E for steel $=$ 30,000,000 lb./sq. inch.

Let P $=$ maximum load in rod.

$$E \;=\; \frac{PL}{A\,e}$$

$$e \;=\; \frac{PL}{AE}$$

$$=\; \frac{P \times 144}{0.7854 \times 30000000}$$

$$=\; 0.00000612\,P$$

$$P \;=\; 2\,W\!\left(\frac{h}{e} + 1\right)$$

$$P \;=\; 2 \times 400\!\left(\frac{2}{0.00000612\,P} + 1\right)$$

$$P \;=\; \frac{1600}{0.00000612\,P} + 800$$

$$0.00000612\,P^2 \;=\; 1600 + 0.004896\,P.$$

$$1\;53\,P^2 - 1224\,P - 400000000 \;=\; 0$$

$$P \;=\; \frac{1224 \pm \sqrt{1498000 + 2448000000}}{3.06}$$

$$=\; \frac{1224 \pm 49430}{3.06}$$

$$=\; \frac{50654}{3.06}$$

$$=\; 16{,}550 \text{ lb.}$$

$$\text{Stress} \;=\; \frac{P}{A}$$

$$=\; \frac{16550}{0.7854}$$

$$=\; 21{,}100 \text{ lb./sq. inch.}$$

TORSION

When one end of a shaft is subjected to a couple at right-angles to its longitudinal axis, there must be a couple on the other end giving an equal and opposite moment, in order to maintain equilibrium. The effect of these couples is to make the shaft twist—*i.e.*, one end will rotate slightly relative to the other end. The moment of the couple is called the twisting moment or torque, and the shaft is said to be subjected to torsion.

Torsional Stress and Strain.

The forces in the material on one side of any normal section will tend to twist the shaft in a clockwise direction, whilst those on the other side will have an equal and opposite tendency to twist it in an anti-clockwise direction. Thus at any portion of the section the two parts of the shaft will exert equal and opposite forces parallel to their surface of contact, and therefore the shaft will be subjected to shear stress.
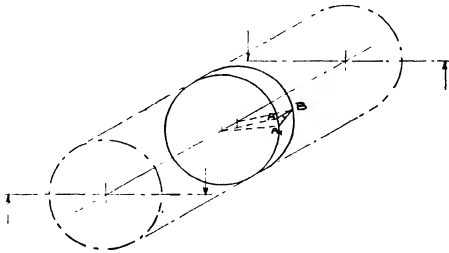
FIG. 83

Consider a small length of such a shaft (Fig. 83). Let AB be a straight line on the surface, parallel to the axis, before the torque is applied. On the application of the torque, A will move to A_1 relative to B. The angle ABA_1 is the angular distortion, and the shear strain at the surface. It will readily be seen that this angle decreases uniformly to zero at the axis of the shaft. This gives the important result that shear strain, and therefore (within the limit of proportionality) the shear stress, is proportional to the distance from the axis.

Strength of Shafts.

Consider the cylindrical bar (Fig. 84) fixed at one end, and at the other having a force P applied at a distance R from the axis $O-O$, and an equal and opposite force P applied at the axis; thus forming a couple, and subjecting the bar to pure torsion.

There is a torque applied to the bar equal to PR, which must be resisted by an equal and opposite moment, called the Moment of Resistance, exerted by the material of the bar.

Consider an elementary ring of radius x_1 and area a_1, and let the stress at this radius be f_1.
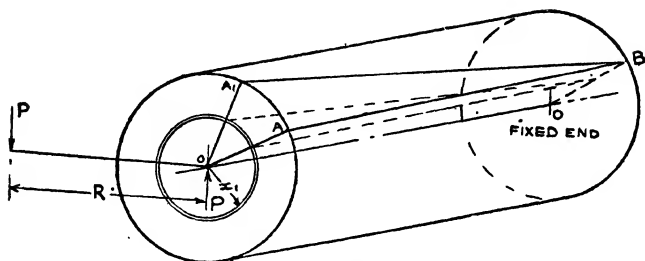
<p style="text-align:center">FIG. 84</p>

Let f_s be the maximum shear stress in the bar—*i.e.*, the stress at the surface—and y the distance of the surface from the axis.

Then, as stress is proportional to distance from axis—

$$\frac{f_1}{f_s} = \frac{x_1}{y}$$

$$f_1 = \frac{f_s x_1}{y}$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

therefore the force in the elementary ring $= f_1 a_1$

and its moment about the axis $= f_1 a_1 x_1$

$$= \frac{f_s}{y} a_1 x_1{}^2.$$

Total moment of resistance $=$ sum of moments of all the elementary rings about the axis, making up the whole cross-section

$$= \frac{f_s}{y} (a_1 x_1{}^2 + a_2 x_2{}^2 + a_3 x_3{}^2 + \text{ etc.})$$

$$= \frac{f_s}{y} \Sigma a\ x^2.$$

But $\Sigma a\ x^2 =$ Moment of Inertia of section about axis $O\text{-}O$.

$\therefore$ Moment of Resistance $= \dfrac{f_s}{y} Ioo$

But Moment of Resistance $=$ Torque.

$$\therefore \text{Torque} = PR = \frac{f_s}{y} Ioo.$$

Ioo, the polar Moment of Inertia, may be found for standard sections in most engineering handbooks.

For a solid cylinder $Ioo = \dfrac{\pi}{32} D^4$

and for a hollow cylinder $Ioo = \dfrac{\pi}{32} (D^4 - d^4)$

where D = outside diameter,

d = inside diameter,

y = will equal $\dfrac{D}{2}$ in each case.

This gives, for a solid cylinder,

$$\text{Torque} = \frac{f_s}{y} Ioo$$

$$= \frac{f_s \pi D^4}{\dfrac{D}{2} \times 32}$$

$$= \frac{\pi}{16} f_s D^3$$

and for a hollow cylinder

$$\text{Torque} = \frac{f_s}{y} Ioo$$

$$= \frac{f_s \pi (D^4 - d^4)}{\dfrac{D}{2} \times 32}$$

$$= \frac{\pi}{16} f_s \frac{(D^4 - d^4)}{D}$$

For a square shaft

$$Ioo = \frac{b^4}{6}$$

$$y = \frac{a}{2} = \frac{b}{\sqrt{2}}$$

where b = length of side of square,

and a = length of diagonal.

$$\text{Torque} = \frac{f_s Ioo}{y}$$

$$= \frac{\sqrt{2} f_s b^4}{6b}$$

$$= \frac{f_s b^3}{3\sqrt{2}} = \frac{f_s a^3}{12}$$

Comparison of Solid and Hollow Shafts.

The stress in a shaft being proportional to the distance from the axis, it follows that with a solid shaft the centre portion is only subjected to a relatively small stress, when the outside is subjected to the maximum permissible stress. In other words, only the outside is being used to the full capacity of the material.

The torque a shaft may withstand is equal to the moment of resistance exerted by the material, when the stress at the outside is equal to the maximum permissible stress. It has been seen that this moment of resistance is equal to the sum of the moments in all the elementary rings of which the shaft may be said to be composed. Also the force in any elementary ring is equal to the product of the stress in that ring and its area.

When the rings are near the axis the stress is small, and therefore the force is also small. The moment will be small because this force is small, and also because the arm is small.

Now consider these rings taken from the centre and placed on the outside, thus forming a hollow shaft. They will now be at points of high stress and the force exerted by them will be increased ; also, being at a greater distance from the axis, the arm will be increased, thus giving a greater moment. It follows that the total moment of resistance will be greater, and a hollow shaft will resist a greater torque than a solid one of the same cross-sectional area.

To illustrate this by an example : find the torque transmitted by

  (a) A solid shaft 4-inch diameter ;
  (b) A hollow shaft 5-inch outside and 3-inch inside diameter—
       i.e., the same cross-sectional area.

Take maximum stress in each case equal to 20,000 lb./sq. inch.

Solid shaft—
$$T = \frac{\pi}{16} f_s D^3$$
$$= 0\cdot196 \times 20000 \times 64$$
$$= \quad 250{,}880 \text{ inch lb.}$$

Hollow shaft—
$$T = \frac{\pi}{16} f_s \frac{(D^4 - d^4)}{D}$$
$$= \frac{0\cdot196 \times 20000}{5} (625 - 81)$$
$$= \frac{0\cdot196 \times 20000 \times 544}{5}$$
$$= 426{,}496 \text{ inch lb.}$$

Torque transmitted by solid shaft $= 250,880$ inch lb.
Torque transmitted by hollow shaft $= 426,496$ inch lb.

Thus it is seen that with the hollow shaft the strength has been nearly doubled, without adding any weight.

Approximate Method for a Thin Tube.

A very near approximation may be made in the case of a tube where the walls are thin compared to the diameter. In this case the material may be considered to be all at a distance equal to the mean radius from the centre, and therefore under a uniform shear stress.

The moment of resistance will equal
$$FR_1 = T,$$
where F is the resistance offered by the material of the tube, and R_1 the mean radius.

The stress being the same all over, we may write
$$\text{Shear stress } f_s = \frac{F}{\text{Cross-sectional area}} = \frac{F}{2\pi R_1 t}$$
$$F = f_s 2\pi R_1 t$$
$$T = FR_1$$

$$\therefore \; T \; = f_s 2\pi R_1{}^2 t$$

$$\text{,} \;\; = f_s \frac{\pi D_1{}^2 t}{2}$$

where $D_1 =$ mean diameter

and $t =$ thickness.

EXAMPLE.—Find the stress in a 4-inch diameter, 0·08-inch thick tube when subjected to a torque of 4,000 in. lb.

By first method—

$$T \; = \; \frac{\pi}{16} f_s \frac{(D^4 - d^4)}{D}$$

$$f_s \; = \; \frac{16 T D}{\pi (D^4 - d^4)}$$

$$= \; \frac{16 \times 4000 \times 4}{\pi (256 - 217 \cdot 5)}$$

$$= \; \frac{16 \times 4000 \times 4}{\pi \times 38 \cdot 5}$$

$$= \; 2,120 \text{ lb./sq. in.}$$

Second approximate method—

$$T \; = f_s \frac{\pi D_1{}^2 t}{2}$$

$$f_s \; = \; \frac{2 T}{\pi D_1{}^2 t}$$

$$= \; \frac{2 \times 4000}{\pi \times 3 \cdot 92^2 \times 0 \cdot 08}$$

$$= \; \frac{8000}{\pi \times 15 \cdot 37 \times 0 \cdot 08}$$

$$= \; 2,070 \text{ lb./sq. inch.}$$

This is only an error of about $2\frac{1}{2}$ per cent.

Angle of Twist.

Referring to Fig. 84, the line AB on the surface, originally straight, becomes A_1B after strain takes place. A_1B everywhere makes an angle equal to ABA_1 with AB. The angle ABA_1 is the shear strain anywhere on the surface.

As angle ABA_1 is small it equals $\dfrac{AA_1}{AB}$ radians.

i.e., Shear strain $= \dfrac{AA_1}{AB}$ radians.

But Shear strain $= \dfrac{\text{Shear stress}}{\text{Modulus of Rigidity}}$

$$\therefore \; \frac{AA_1}{AB} = \frac{f_s}{C}$$

and $AA_1 = \dfrac{f_s l}{C}$

where $l =$ length of shaft $= AB$.

The angle A_1OA is called the Angle of Twist, and is the amount the length of shaft AB twists when subjected to the torque PR.

If θ is the angle of twist in radians,

$$\theta = \frac{AA_1}{D/2}$$

Substituting $\frac{f_s l}{C}$ for AA_1,

$$\theta = \frac{2f_s l}{DC} \text{ radians}$$

$$= \frac{360 \, f_s l}{\pi DC} \text{ degrees.}$$

Substituting for f_s, we have for a solid shaft—

$$T = \frac{\pi}{16} f_s D^3$$

$$f_s = \frac{16T}{\pi D^3}$$

$$\theta = \frac{360 \times 16 \times T \times l}{\pi^2 D^4 C}$$

$$= \frac{583 \, Tl}{D^4 C} \text{ degrees.}$$

Combined Torsion and Bending.

It often happens that a torque is applied to a shaft by a single force instead of a couple. Consider the case of the shaft (Fig. 85) fixed at A and having a single force P applied at a distance R from the axis at B. The shaft will have to transmit a torque PR and the unbalanced force P to the fixed end. The effect of this will be to bend the shaft as well as to twist it. The tendency of P to bend the shaft is called the Bending Moment.
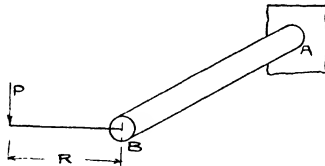
FIG. 85

The method of finding the bending moment is given in Chapter X.

If a shaft is subjected to a torque T and a bending moment M, they will together produce an equivalent torque T_1 such that

$$T_1 = \sqrt{M^2 + T^2}.$$

Power Transmitted by Shafts.

If power is transmitted by a shaft, the product of the torque T and the angle turned through per minute measured in radians gives the work transmitted per minute. There are 2π radians in one revolution, therefore if a shaft is running at N r.p.m., it turns through $2\pi N$ radians per minute, and the work transmitted per minute $= T 2\pi N$.

Horse-power equals work in ft. lb. per minute, divided by 33,000.
Thus horse-power transmitted by a shaft

$$= \frac{T \, 2\pi \, N}{33000}$$

$$T = \frac{\text{H.P.} \times 33000}{2\pi N} \text{ ft. lb.}$$

$$= \frac{\text{H.P.} \times 33000 \times 12}{2\pi N} \text{ inch lb.}$$

$$= \frac{63000 \text{ H.P.}}{N} \text{ inch lb.}$$

Shaft Couplings.

Joints are often made in shafts by means of flange couplings, such as that shown in Fig. 86. Suppose a torque T is being transmitted by the shaft. There will be a shear force in each bolt such that—

$$T = FRn,$$

where F = shear force in each bolt,

R = radius of bolt circle,

n = number of bolts.
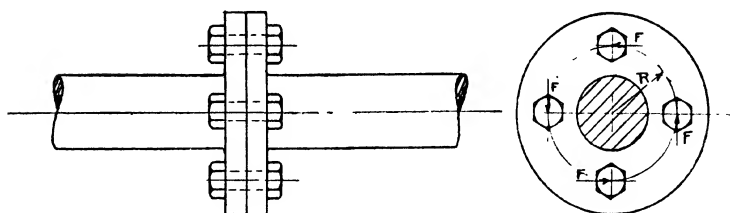
FIG. 86

In the case considered the bolts are in single shear.

$$\text{Shear stress} = f_s = \frac{F}{\frac{\pi}{4} d^2}$$

$$F = f_s \frac{\pi}{4} d^2$$

$$\therefore \ T = f_s \frac{\pi}{4} d^2 Rn.$$

In the case of the airscrew hub (Fig. 87), the torque is transmitted to the airscrew by two flanges and the bolts are in double shear.

$$\text{Thus } F = f_s \frac{\pi}{2} d^2$$

$$\text{and } T = f_s \frac{\pi}{2} d^2 Rn.$$
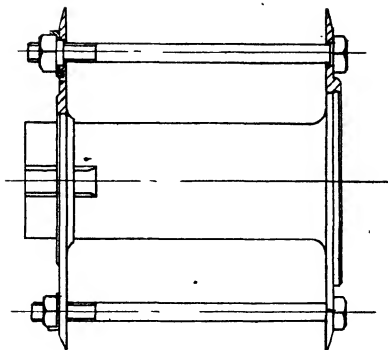
FIG. 87

For wooden airscrews it is usual practice to rough machine the inner surfaces of the flanges, so that when the bolts are tightened there is sufficient friction grip between the flanges and airscrew to transmit the torque. This prevents the load being transmitted by the bolts bearing on the wood, and the bolts will only be subject to tension.

EXAMPLE 1.—A solid shaft, 3 in. diameter, transmits a torque of 4,500 ft. lb. It is replaced by a similar shaft of $3\frac{1}{2}$-inch diameter. If the maximum stress remains the same, what torque does it now transmit ?

$$T = \frac{\pi}{16} f_s D^3$$

$$T \propto D^3$$

$$\therefore \text{Torque required} = \frac{4500 \times 3 \cdot 5^3}{3 \cdot 0^3}$$

$$= \frac{4500 \times 42 \cdot 875}{27}$$

$$= 7,146 \text{ ft. lb.}$$

EXAMPLE 2.—The control lever on an aileron spar is 9 inches from centre line of control cable to centre of spar. The load in the cable is 400 lb. If the spar is a steel tube of 2 inches diameter and 0·08 inch thick, find the torsional stress in the spar, and the angle of twist on a length of 18 inches. Take $C = 12,000,000$ lb./sq. inch.

$$T = PR = \frac{\pi}{16} f_s \frac{(D^4 - d^4)}{D}$$

$$\text{Stress } f_s = \frac{PR \times 16 \times D}{\pi(D^4 - d^4)}$$

$$= \frac{400 \times 9 \times 16 \times 2}{\pi(2^4 - 1 \cdot 84^4)}$$

$$= \frac{400 \times 9 \times 16 \times 2}{\pi \times 4 \cdot 53}$$

$$= 8,100 \text{ lb./sq. inch.}$$

Angle of Twist $\theta = \dfrac{360 \, f_s l}{\pi DC}$

$$= \dfrac{360 \times 8100 \times 18}{\pi \times 2 \times 12000000}$$

$$= 0\cdot7 \text{ degrees.}$$

EXAMPLE 3.—A solid shaft is subjected to a torque of 2,400 ft. lb., and a bending moment of 2,000 ft. lb.

The ultimate shear stress is 30 tons/sq. inch, and a factor of safety of 6 is required.

Find the equivalent torque and a suitable diameter for the shaft.

Equivalent torque $= T_1 = \sqrt{M^2 + T^2}$

$$= \sqrt{2000^2 + 2400^2}$$

$$= \sqrt{9760000}$$

$$= 3,124 \text{ ft. lb.}$$

$$= 3124 \times 12$$

$$= 37,488 \text{ inch lb.}$$

$f_s = \dfrac{30 \times 2240}{6} = 11200 \text{ lb./sq. inch}$

$T = \dfrac{\pi}{16} f_s D^3$

$D = \sqrt[3]{\dfrac{16 \, T}{\pi f_s}}$

$$= \sqrt[3]{\dfrac{16 \times 37488}{\pi \times 11200}}$$

$$= \sqrt{17\cdot04}$$

$$= 2\cdot57 \text{ inch}$$

or $2\frac{5}{8}$ inch diameter shaft.

EXAMPLE 4.—Find a suitable diameter for a solid shaft that has to transmit 200 h.p. at 1,200 r.p.m. The stress in the shaft is not to exceed 6 tons/sq. inch.

$T = \dfrac{63000 \text{ H.P.}}{N}$

$$= \dfrac{63000 \times 200}{1200}$$

$$= 10,500 \text{ inch lb.}$$

$T = \dfrac{\pi}{16} f_s D^3$

$$D = \sqrt[3]{\frac{16T}{\pi f_s}}$$

$$= \sqrt[3]{\frac{16 \times 10500}{\pi \times 6 \times 2240}}$$

$$= \sqrt[3]{3.98}$$

$$= 1\cdot58 \text{ inch}$$

$$\text{or } 1\tfrac{5}{8} \text{ inches.}$$

EXAMPLE 5.—The maximum horse-power which a certain aero engine can develop is 375 at a speed of 1,700 r.p.m.   The power is transmitted to the airscrew by 6 bolts $\frac{1}{2}$ inch diameter passing through the boss and two flanges in the airscrew hub, so that the bolts are in double shear.   The radius of the bolt circle is 3 inches.

Find the torque transmitted to the airscrew, and the shear stress in the bolts.   Max. torque equals 120 per cent. mean torque.

$$\text{Max. } T = \frac{63000 \text{ H.P.} \times 1\cdot20}{N}$$

$$= \frac{63000 \times 375 \times 1\cdot20}{1700}$$

$$= 16,680 \text{ inch lb.}$$

$$T = f_s \frac{\pi}{2} d^2 R n$$

$$f_s = \frac{2T}{\pi d^2 R n}$$

$$= \frac{2 \times 16680 \times 2 \times 2}{\pi \times 1 \times 1 \times 3 \times 6}$$

$$= 2,360 \text{ lb./sq. inch.}$$

EXAMPLE 6.—A solid steel shaft 2 inch diameter is 16 inches long and transmits a torque of 3,000 ft. lb.   Find the angle of twist.   Take $C = 12,000,000$ lb./sq. inch.

$$\theta = \frac{583 \, Tl}{D^4 C}$$

$$= \frac{583 \times 3000 \times 12 \times 16}{16 \times 12000000}$$

$$= 1\cdot75 \text{ degrees.}$$

Close-coiled Helical Springs.

When a helical spring is loaded axially, and has its coils so close together that they lie very nearly perpendicular to the axis, the material may be considered as subjected to torsion only.

Consider the spring (Fig. 88) subjected to a tensile load P, and let the mean radius of the coils be R.   At any section S the material

resists a torque PR. If the spring is made of wire of circular cross-section, of which the diameter is d, its moment of resistance will be the same as that of a solid cylinder of diameter d—

$$i.e., \quad \frac{\pi}{16} f_s d^3$$

Therefore for the spring

$$PR = \frac{\pi}{16} f_s d^3$$

or maximum stress in the spring

$$= f_s = \frac{16\ PR}{\pi d^3}$$
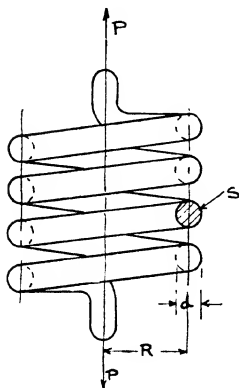
Fig. 88
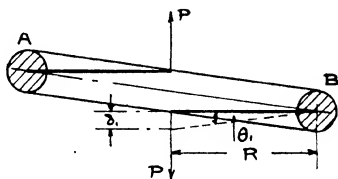
Fig. 89

Deflection of Spring.

Consider any half coil (Fig. 89) and let it be assumed to twist through an angle θ^1 under the axial load P. If the end A is fixed, the free end B will have an axial movement δ_1 equal to $R\theta^1$, due to the twist θ^1.

In the same way the whole spring will have an axial movement or deflection due to the total twist.

Let the angle of twist for the whole spring $= \theta$, the total deflection $= \delta$ and the number of coils $= n$.

$$\text{Then} \quad \theta = \frac{2 f_s l}{dC} \text{ radians, as for a shaft,}$$

$$\text{Length } l = 2\pi R n.$$

Substituting for l

$$\theta = \frac{4\pi f_s R n}{dC}$$

Deflection $\delta = R\theta = \dfrac{4\ \pi f_s R^2 n}{dC}$

Substituting $\dfrac{16\ PR}{\pi d^3}$ for f_s

$$\delta = \frac{64\ P R^3 n}{d^4\ C}$$

Volume of Spring.

Volume of spring $= V =$ Cross-sectional area $\times l$

$$= \frac{\pi}{4}d^2 \times 2\pi Rn$$

$$= \frac{\pi^2}{2} d^2 Rn$$

From above, $\delta = \dfrac{4\pi f_s R^2 n}{dC}$

$$P\delta = \frac{4\pi f_s P R^2 n}{d C}$$

Substitute $\dfrac{\pi}{16}f_s d^3$ for PR

$$P\delta = \frac{4\pi^2 f_s^2 d^3 Rn}{16dC}$$

$$= \frac{f_s^2 \pi^2 d^2 Rn}{4C}$$

$$= \frac{f_s^2 V}{2C}$$

$$V = \frac{2P\delta C}{f_s^2}$$

The above formulæ also apply when the spring is in compression.

When the spring is not close-coiled, the axial force causes bending as well as torsion, but in most cases the torsional stress is so much greater than the bending stress that the latter may be neglected.

EXAMPLE 1.—A close-coiled helical spring has a mean diameter of 3 inches. It is made of round steel wire of $\frac{1}{4}$ inch diameter, and there are 6 coils. Find the maximum load which may be applied if the permissible stress is limited to 45,000 lb./sq. inch, also the deflection under a load of 40 lb.

C for steel $= 12,000,000$ lb./sq. inch.

$$f_s = \frac{16 PR}{\pi d^3}$$

$$P = \frac{f_s \pi d^3}{16R}$$

$$= \frac{45000 \times \pi}{16 \times 1\cdot5 \times 4 \times 4 \times 4}$$

$$= 91\cdot9 \text{ lb. maximum load.}$$

$$\delta = \frac{64 P R^3 n}{d^4 C}$$

$$= \frac{64 \times 40 \times 1\cdot5 \times 1\cdot5 \times 1\cdot5 \times 6}{0\cdot25 \times 0\cdot25 \times 0\cdot25 \times 0\cdot25 \times 12000000}$$

$$= 1\cdot1 \text{ inch deflection.}$$

EXAMPLE 2.—A steel spring is required to compress $\frac{1}{8}$ inch for every 100 lb. up to 200 lb. Design the spring if the maximum stress is not to exceed 60,000 lb. sq./inch.

$$f_s = \frac{16\,P\,R}{\pi d^3}$$

$$R = \frac{f_s\,\pi\,d^3}{16P}$$

$$= \frac{60000 \times \pi \times d^3}{16 \times 200}$$

$$= 58.9\,d^3.$$

$$V = \frac{2P\delta C}{f_s^2}$$

$$= \frac{2 \times 200 \times 1 \times 12000000}{60000 \times 60000}$$

$$= \frac{4}{3}\ \text{cu. inch.}$$

$$V = \frac{\pi^2}{2}\,d^2 R n$$

$$n = \frac{2V}{\pi^2 d^2 R}$$

$$= \frac{2 \times 4}{3 \times \pi^2 \times d^2 \times 58.9\,d^3}$$

$$= \frac{0.0046.}{d^5}$$

Now find the values of R and n for different values of d, until a spring with a suitable number of coils is found, such that it is not too long nor the coils so few that the deflection of each coil is excessive.

d inch.	R inch $= 58.8\,d^3$	$n = \dfrac{0.0046}{d^5}$
0·4	3·76	0·46
0·3	1·59	1·9
0·2	0·47	14·4
0·25	0·92	4·7

The $\frac{1}{4}$ inch diameter wire spring is suitable.

Answer. $\frac{1}{4}$ inch diameter round wire.
           1·84 inch mean diameter of coils.
           4·7 coils.

CHAPTER X

BEAMS

Any structure acted upon by forces oblique to its longitudinal axis is called a Beam. If these forces are perpendicular to the longitudinal axis there will be a bending and shearing effect produced which will vary along the length of the beam.

If the forces are not perpendicular to the axis, their perpendicular components will produce a bending and shearing effect, and their longitudinal components will produce an additional bending effect together with extension or contraction of the beam as a whole.
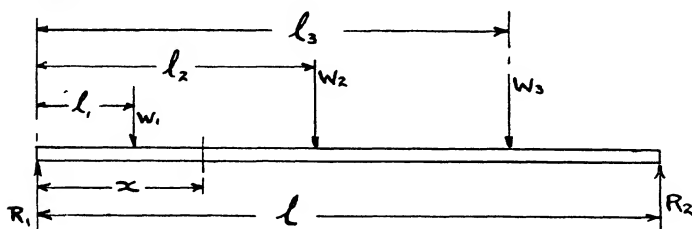
FIG. 90

We will first consider beams having all their external forces perpendicular to the longitudinal axis. Take the case of the beam in Fig. 90. As the beam is at rest, it must be in equilibrium under the
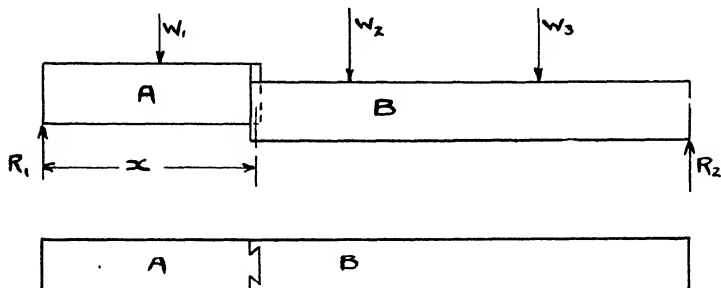
FIG. 91

action of all the external forces—*i.e.*, the algebraic sum of all the forces equals zero and the algebraic sum of the moments of all the forces about any point equals zero.

Neglecting the weight of the beam,

$$W_1 + W_2 + W_3 - R_1 - R_2 = 0.$$

Taking moments about the support R_1,

$$l_1 W_1 + l_2 W_2 + l_3 W_3 - l R_2 = 0.$$

From these two equations the reactions at the supports R_1 and R_2 may be found.

Shearing Force.

Consider a section of the beam between the loads W_1 and W_2 distance x from R_1. Imagine the beam is dovetailed at this section as illustrated in Fig. 91. It will readily be seen that the reaction R_1 is tending to slide A up relative to B, and the load W_1 is tending to slide A down. There will be a resultant tendency for A to slide up equal to $R_1 - W_1$. On the portion B there will be a tendency for B to slide down equal to $W_2 + W_3 - R_2$. The tendency for A to slide up will be equal to the tendency for B to slide down, for

$$W_1 + W_2 + W_3 - R_1 - R_2 = 0,$$

$$\text{therefore } W_2 + W_3 - R_2 = R_1 - W_1.$$

This sliding tendency is a shearing tendency which the material of the beam must be strong enough to resist. The shearing force, which in this case equals $R_1 - W_1 = W_2 + W_3 - R_2$, is the resultant force perpendicular to the beam on A or B. It follows that the shear force at any section of a beam is equal to the algebraic sum of the forces perpendicular to the beam on either side of the section considered.

EXAMPLE.—Find the shear force at the section X in the beam shown in Fig. 92.
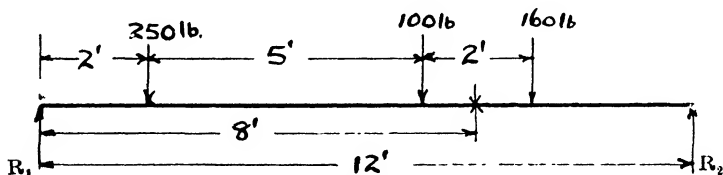
FIG. 92

To find the reactions take moments about R_1.

$$2 \times 250 + 7 \times 100 + 9 \times 160 - 12R_2 = 0.$$

$$12R_2 = 500 + 700 + 1440.$$

$$R_2 = \frac{2640}{12} = 220 \text{ lb.}$$

$$250 + 100 + 160 - 220 - R_1 = 0.$$

$$R_1 = 290 \text{ lb.}$$

Considering the forces to the left of Section X—

$$\text{Shear force} = 290 - 250 - 100$$

$$= -60 \text{ lb.}—i.e., 60 \text{ lb. downwards.}$$

On the right of Section X—

$$\text{Shear force} = 220 - 160$$

$$= 60 \text{ lb.}—i.e., 60 \text{ lb. upwards.}$$

Bending Moment.

Now consider the beam to be hinged at the section distance x from R_1, with a hinge that is sufficiently tight to prevent turning, as shown in Fig. 93. The reaction R_1 is tending to turn the portion A in a clockwise direction relative to B, and the load W_1 is tending to turn it in an anti-clockwise direction. The turning effect of a force about any point is the moment of that force about that point—*i.e.*, the product of the force and the perpendicular distance of its line of action from that point.
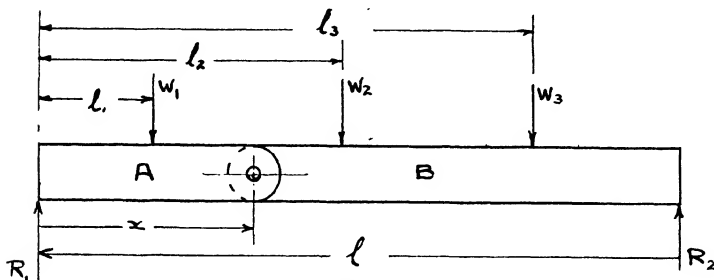
FIG. 93

Thus the turning effect at the hinge in a clockwise direction equals

$$R_1 x - W_1(x - l_1).$$

The forces on the portion B will exert a turning effect equal to

$$W_2(l_2 - x) + W_3(l_3 - x) - R_2(l - x).$$

That these two turning effects are equal and opposite may be seen by taking moments about the hinge and applying the equilibrium condition for the whole beam—viz.—

$$R_1 x - W_1(x - l_1) + W_2(l_2 - x) + W_3(l_3 - x) - R_2(l - x) = 0.$$

Therefore

$$R_1 x - W_1(x - l_1) = -\left[W_2(l_2 - x) + W_3(l_3 - x) - R_2(l - x) \right]$$

—*i.e.*, clockwise turning effect of A = anti-clockwise turning effect of B.

If we now consider the beam not hinged, but in one piece, this turning effect or moment still remains and will cause a straining action called bending. If we imagine the beam is a plank supported at the ends, and the three loads are men standing on it, we can readily picture this bending.

This turning moment is called a Bending Moment in dealing with beams. The material of the beam must be strong enough to resist this moment, by exerting an equal and opposite moment, called the Moment of Resistance, formed by a tensile reaction on the convex side and a compressive reaction on the concave side of the beam at the section.

The bending moment varies along the beam and for the section considered equals—

$$R_1 x - W_1(x - l_1) = W_2(l_2 - x) + W_3(l_3 - x) - R_2(l - x),$$

which is the resultant moment of the forces on A or B about the section. It follows that the bending moment at any section of a beam is equal to the algebraic sum of the moments (about the section) of all the forces on either side of the section considered.

EXAMPLE.—Find the bending moment at the section X in the beam shown in Fig. 92.

The reactions have already been found for the previous example on shear force—*i.e.,*

$$R_1 = 290 \text{ lb.,}$$
$$R_2 = 220 \text{ lb.}$$

Considering the forces to the left of the section, and calling clockwise moments positive—

Bending Moment $= 290 \times 8 - 250 \times 6 - 100 \times 1$

$= 2320 - 1500 - 100$

$= 720 \text{ ft. lb.}$

As the bending moment is equal to the sum of the moments on *either* side of the section, the same answer will be obtained by considering the forces to the right of the section : thus

Bending Moment $= 160 \times 1 - 4 \times 220$

$= 160 - 880$

$= -720 \text{ ft. lb.}$

It will be noticed that these two answers are equal but of opposite sign—*i.e.,* the section X is subjected to a clockwise moment from the left and an equal anti-clockwise moment from the right. This must be the case from the conditions of equilibrium already explained.

Signs.

Whether the bending moment is clockwise or anti-clockwise on one side of the section makes no difference to the bending effect ; for if the beam is viewed from the other side a clockwise moment becomes an anti-clockwise moment.

It will, however, be convenient always to call a clockwise moment from the left and an anti-clockwise moment from the right positive, for the following reason. The forces acting on the beam will bend it so that it either becomes concave or convex on the top. By using the signs given a positive bending moment will always show the beam to be concave on top, and a negative bending moment will show it to be convex on top, at the section considered. It follows that where the bending moment is positive, the top of the beam will be in compression, and the bottom in tension, and *vice versa*. Figs. 94 (*a*) and (*b*) show positive and negative bending respectively.

(a)   (b)
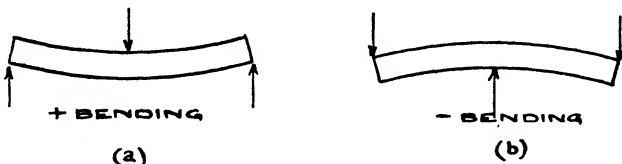
FIG. 94

Again, in dealing with shearing forces, if an upward force from the left and a downward force from the right is called positive, a positive shearing force will always show that the portion of the beam to the left of the section considered is tending to ascend relative to the portion

to the right, and *vice versa.* Figs. 95 (*a*) and (*b*) show positive and negative shearing respectively.

The system of signs given above will be used throughout this book, but, unfortunately, there is no official standard, so that in reading other books the student is quite likely to find the signs have been reversed.
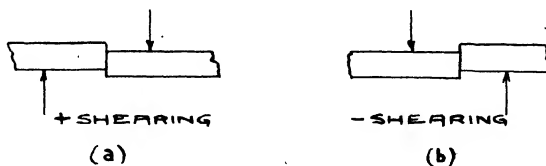
FIG. 95

Bending Moment and Shearing Force Diagrams.

As both the bending moment and shearing force will vary along the length of a beam, it is convenient to plot graphs showing this variation.

If the values of bending moment and shearing force be plotted against the span, two graphs will be obtained which will show the bending moment and shearing force at any section of the beam. Such graphs are called Bending Moment and Shearing Force Diagrams.

We will consider some typical cases for simply loaded beams. Numerical examples are taken, as the author has found that if symbols are used, students learn the results as formulæ, and are apt then to use them when they do not apply. Also the object should not be to learn special cases, but to understand the principles sufficiently to apply them to any case.

Cantilevers.

Cantilevers are beams rigidly fixed at one end, and free at the other.

EXAMPLE 1. (Fig. 96).—Cantilever 12 feet long with a load of 4 tons at the free end. Neglect the weight of the beam.

In dealing with cantilevers always work from the free end, as the forces at the fixed end are unknown.

In this case the shearing force (S.F.) will be —4 tons for the whole length of the beam.
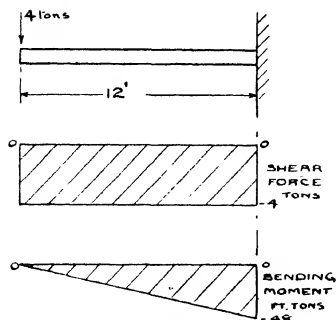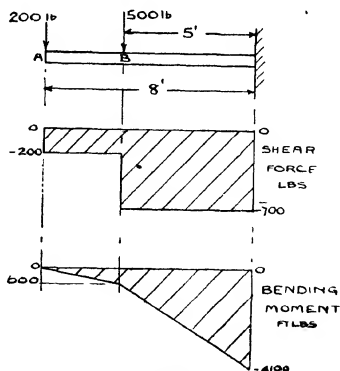
FIG. 96   FIG. 97

The bending moment (B.M.) will be zero at the extreme left, for there are no forces and therefore no moment to the left of this section. At the fixed end

$$\text{B.M.} = -4 \times 12 = -48 \text{ ft. tons.}$$

At any distance x from the free end,

$$\text{B.M.} = -4 \times x \text{ ft. tons}$$

—*i.e.*, the B.M. is proportional to the distance along the beam from the free end. The diagram will thus be a straight line from zero at the free end to 48 ft. tons at the fixed end.

EXAMPLE 2 (Fig. 97).—Cantilever 8 ft. long, with a load of 200 lb. at the free end, and a load of 500 lb. 5 feet from the fixed end.

The S.F. will be −200 lb. between A and B. It will be increased by −500 lb. to −700 lb. between B and the fixed end.

The B.M. will be zero at A, and from there increase uniformly to $-3 \times 200 = -600$ ft. lb. at B. From B the B.M. will again increase uniformly to its value at the fixed end—viz.,

$$-8 \times 200 - 5 \times 500 = -1600 - 2500$$
$$= -4,100 \text{ ft. lb.}$$

It may here be noted that when the loads are applied at fixed points only, the B.M. will vary uniformly between the loads, and the B.M. diagram may be drawn by obtaining the B.M.s at the points of application of the loads and joining them by a straight line. This does not apply where there are distributed loads on the beam.

EXAMPLE 3 (Fig. 98).—Cantilever 12 feet long with a load of 4 tons evenly distributed over the whole length.

There will be $\frac{1}{3}$ ton per foot length.

The S.F. will be zero at the free end, there being no forces to the left. At 1 foot inwards there will be $\frac{1}{3}$ ton to the left, and the S.F. will therefore be $-\frac{1}{3}$; at 2 feet it will be $-\frac{2}{3}$ ton ; at 3 feet it will be −1 ton, and so on, uniformly increasing with the distance from the free end until it reaches a maximum equal to the total load of 4 tons at the fixed end.

The B.M. will be zero at the free end, but will not increase uniformly. At any section distance x from the free end, there will be a force of (load per foot) $\times x = \frac{1}{3}x$ tons to the left of the section. This force will act at the C.G. of the portion of load—*i.e.*, at distance $\frac{x}{2}$ from the section considered. Thus the B.M. at any section distance x feet from the free end will equal

$$-\frac{1}{3}x \times \frac{x}{2} = -\frac{x^2}{6} \text{ ft. tons.}$$

As the B.M. is proportional to distance squared, the B.M. curve will be a parabola.

At 6 feet from the free end the B.M. equals—

$$-\frac{1}{3} \times 6 \times \frac{6}{2} = -6 \text{ ft. tons.}$$

At the fixed end the B.M. is a maximum and equal to—

Total load $\times$ half the length
$$= -4 \times 6 = -24 \text{ ft. tons.}$$
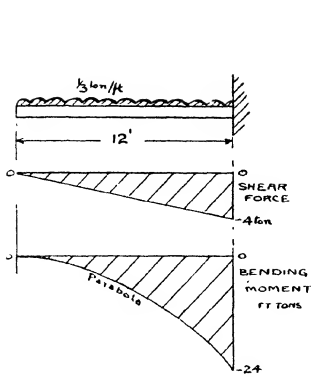
FIG. 98
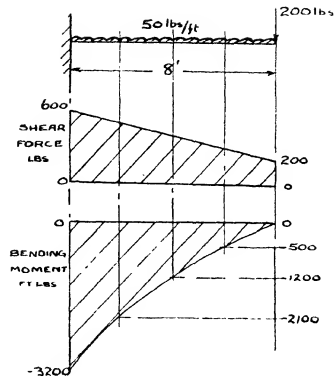
FIG. 99

EXAMPLE 4 (Fig. 99).—Cantilever 8ft. long with a concentrated load of 200 lb. at the free end and a distributed load of 50 lb. per foot on the whole length.

The S.F. will start at + 200 lb. at the free end and increase uniformly at the rate of 50 lb./ft. to + 600 lb. at the fixed end.

The B.M. will be zero at the free end, and at any distance x from the free end will be $-200\,x - 50\,x \times \dfrac{x}{2}$

$$\text{Thus at 2 ft. from free end B.M.} = -200 \times 2 - 50 \times 2 \times 1$$
$$= -500 \text{ ft. lb.}$$
$$\text{at 4 ft. from free end B.M.} = -200 \times 4 - 50 \times 4 \times 2$$
$$= -1200 \text{ ft. lb.}$$
$$\text{at 6 ft. from free end B.M.} = -200 \times 6 - 50 \times 6 \times 3$$
$$= -2100 \text{ ft. lb.}$$

and at the fixed end the B.M. is a maximum and equal to
$$-200 \times 8 - 50 \times 8 \times 4 = 3,200 \text{ ft. lb.}$$

Simply Supported Beams.

EXAMPLE 5 (Fig. 100).—12 ft. beam freely supported at the ends, with a load of 4 tons in the centre.
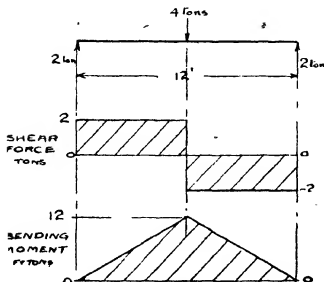
FIG. 100
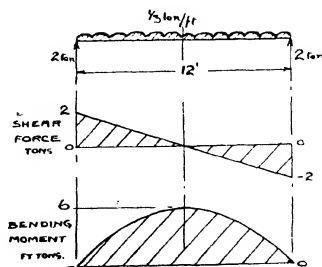
FIG. 101

The beam being symmetrical, the load on each support will be 2 tons.
S.F. between left support and middle   = 2 tons.
S.F. between middle and right support = 2−4 = −2 tons.

B.M. at left support = 0

Maximum B.M. is at centre=Reaction at support × distance from support to centre

$$= 2 \times 6 = 12 \text{ ft. tons.}$$

B.M. at right support = 0.

EXAMPLE 6 (Fig. 101).—12 ft. beam freely supported at the ends, with a load of 4 tons evenly distributed over the whole length.

Reactions at supports = 2 tons each.

Distributed load of $\frac{1}{3}$ ton per foot.

S.F. at left support 2 tons, decreasing at the rate of $\frac{1}{3}$ ton per foot to zero at the centre, and −2 tons at the right support.

B.M. at left support = 0.

B.M. at any distance x from left support
= Moment of reaction at support −moment of length x of distributed load

$$= 2x − \frac{x}{3} \times \frac{x}{2}$$

$$= 2x − \frac{x^2}{6}$$

Maximum B.M. at centre $= 2 \times 6 − \dfrac{36}{6} = 12 − 6 = 6$ ft. tons.

The rest of the curve may be found by taking two or three intermediate sections.

EXAMPLE 7 (Fig. 102).—Beam supported at ends with two concentrated loads.

First find the reactions at the supports.
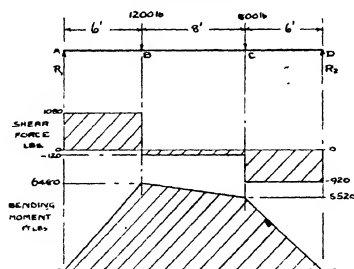
FIG. 102

Taking moments about A,

$$6 \times 1200 + 14 \times 800 − 20\ R_2 = 0.$$

$$R_2 = \frac{7200 + 11200}{20}$$

$$= 920 \text{ lb.}$$

$$1200 + 800 − 920 − R_1 = 0.$$

$$R_1 = 1,080 \text{ lb.}$$

S.F. between A and B = 1080 lb.

S.F. between B and C = 1080−1200 = −120 lb.

S.F. between C and D = 1080−1200−800 = −920 lb.

B.M. at A = 0.

B.M. at B = 6×1080 = 6480 ft. lb.

B.M. at C = 14×1080−8×1200 = 5520 ft. lb.

B.M. at D = 0.

EXAMPLE 8.—Beam with distributed and concentrated loads, supported along the span (Fig. 103).
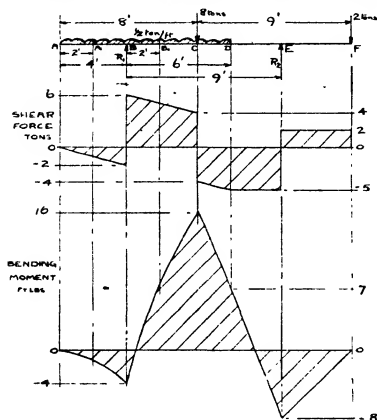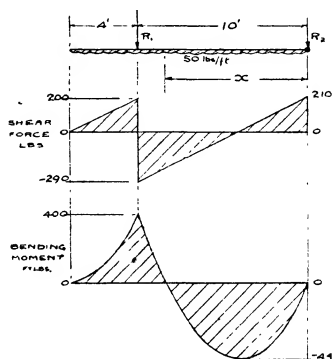
FIG. 103   FIG. 104

Take moments about B to find reactions—

$$1 \times \tfrac{1}{2} \times 10 + 8 \times 4 + 13 \times 2 - 9R_2 = 0.$$

$$R_2 = \frac{5+32+26}{9} = 7$$

$$\tfrac{1}{2} \times 10 + 8 + 2 - 7 - R_1 = 0$$

$$R_1 = 8.$$

S.F. between A and B = 0 decreasing to −2.

S.F. between B and C = 6 decreasing to 4.

S.F. between C and D = −4 decreasing to −5.

S.F. between D and E = −5.

S.F. between E and F = +2.

The B.M. between A and D will be a curve and several intermediate points should be taken.

B.M. at A = 0.

B.M. at A_1 = −2×$\tfrac{1}{2}$×1 = −1.

B.M. at B = −4×$\tfrac{1}{2}$×2 = −4.

B.M. at B_1 = 8×2−6×$\tfrac{1}{2}$×3 = 7.

B.M. at C = 8×4−8×$\tfrac{1}{2}$×4 = 16.

B.M. at D = 8×6−8×2−10×$\tfrac{1}{2}$×5 = 7.

B.M. at E = 8×9−8×5−10×$\tfrac{1}{2}$×8 = −8.

B.M. at F = 8×13−8×9−10×$\tfrac{1}{2}$×12+7×4 = 0.

The B.M. at F or any free end is always zero, but it is as well to work it out as a check. If it should not work out to zero, probably there is a mistake in the reactions.

EXAMPLE 9 (Fig. 104).—A bottom spar of a single bay biplane pin-jointed to the fuselage at one end and supported by a pin-jointed vertical strut at a distance of 10 feet from the fuselage joint. The total length of the spar is 14 feet. Draw the B.M. and S.F. diagrams, and find the maximum B.M. and S.F., assuming there is a lift force evenly distributed over the whole length at the rate of 50 lb. per foot.

Take moments about fuselage joint to find reactions.

$$50 \times 14 \times 7 - 10R_1 = 0$$

$$R_1 = \frac{50 \times 14 \times 7}{10}$$

$$= 490 \text{ lb. downwards.}$$

$$50 \times 14 - 490 - R_2 = 0$$

$$R_2 = 210 \text{ lb. downwards.}$$

The B.M. and S.F. for different points are shown in the table. From the graphs it is found that the maximum B.M. equals 441 ft. lb. at 4·2 ft. from the fuselage, and the maximum S.F. equals 290 lb. at the strut.

Position Distance from Outer End.	S.F., lb.	B.M., ft. lb.
0 ft.	0	0
2 ft.	$50 \times 2 = 100$	$50 \times 2 \times 1 = 100$
4 ft.	Changes from $50 \times 4 = 200$ to $50 \times 4 - 490 = -290$	$50 \times 4 \times 2 = 400$
6 ft.	$50 \times 6 - 490 = -190$	$50 \times 6 \times 3 - 490 \times 2 = -80$
8 ft.	$50 \times 8 - 490 = -90$	$50 \times 8 \times 4 - 490 \times 4 = -360$
10 ft.	$50 \times 10 - 490 = 10$	$50 \times 10 \times 5 - 490 \times 6 = -440$
12 ft.	$50 \times 12 - 490 = 110$	$50 \times 12 \times 6 - 490 \times 8 = -320$
14 ft.	$50 \times 14 - 490 = 210$	$50 \times 14 \times 7 - 490 \times 10 = 0$

Points of Inflection.

In Examples 8 and 9 the value of B.M. changes sign, and as the curve of B.M. is continuous, this involves passing through a zero value ; the point where this occurs is called a Point of Inflection. At this point the bent beam will change from convexity to concavity, or *vice versa.*

The points of inflection may be found from the diagram, or more accurately by equating the B.M. to zero. Take the case of the beam in Fig. 105.

Let x equal the distance of the point of inflection from the end.

B.M. at point of inflection $= 0$

$$5(x-2)-2x = 0$$

$$5x-10-2x = 0$$

$$x = \frac{10}{3} = 3\tfrac{1}{3} \text{ feet.}$$
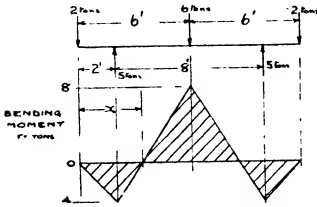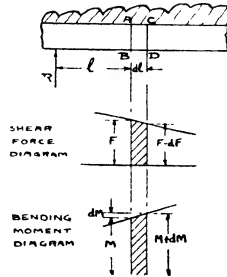
Again, find the distance of the point of inflection from the fuselage oint of the spar in Example 9.

Let $x = $ distance required.

$$50x \times \frac{x}{2} - 210x = 0$$

$$25x^2 - 210x = 0$$

$$25x - 210 = 0$$

$$x = \frac{210}{25} = 8{\cdot}4 \text{ feet.}$$

Relation between Bending Moment and Shearing Force.

Let AB (Fig. 106) be a section of a beam loaded in any way, and the shearing force at this section $=F$ and the bending moment $=M$. Take another section CD an indefinitely small distance dl from AB, and let the S.F. at $CD=F-dF$ and the B.M. $=M+dM$.

Let R be the resultant of all the forces to the left of AB, and let it act at a distance l from AB.

Then $R=F$, and $Rl=M$.

B.M. at $CD = M+dM = R\,(l + dl) - \dfrac{dF \times dl}{2}$

$$M+dM = Rl+R\,.\,dl - \frac{dF \times dl}{2}$$

$$M+dM = M+F\,.\,dl - \frac{dF \times dl}{2}$$

$$dM = F\,.\,dl - \frac{dF \times d}{2}$$

$$\frac{dM}{dl} = F - \frac{dF}{2}$$

dF is the difference between the S.F. at AB and at CD, or the resultant of the loads between AB and CD, since dl is indefinitely small dF will be so small that it may be neglected, and we can write

$$\frac{dM}{dl} = F$$

—*i.e.*, the shearing force F at AB is equal to $\dfrac{dM}{dl}$, which is the slope of the bending moment diagram at AB. It follows that the slope of the bending moment diagram at any section of a beam is equal to the shearing force at that section.

This result is useful in determining the maximum bending moment. Where the B.M. reaches a maximum value its slope will be zero, and this point will occur where the S.F. is zero. Note : when the S.F. changes sign at a concentrated load it passes through a zero value, though it is not apparent on the diagram. Actually no load can be concentrated at a point, but must be distributed over a small length, so that the shearing force diagram at a "concentrated" load is something like that shown in Fig. 107.

Referring to Example 9 (Fig. 104), the section of maximum B.M. will be found from the shearing force diagram to occur either at the strut or 4·2 feet from the fuselage joint. If, as is often the case, only the maximum B.M. is required, it may be found by simply working out the B.M. at these two sections and taking the larger. No bending moment diagram need be drawn.

Again, as $\dfrac{dM}{dl} = F$, $dM = Fdl$—*i.e.*, the difference in B.M. at two sections indefinitely near to each other is equal to the area of the shearing force diagram between these two sections. The difference between the B.M. at any two sections will equal the sum of all the increments dM between the two sections, and the area of the shearing force diagram between the two sections will equal the sum of all the increments of area $F \cdot dx$ between the two sections—*i.e.*,

$$\Sigma dM = \Sigma Fdl.$$

Thus the difference between the B.M. at any two sections is equal to the area of the shearing force diagram between these two sections.
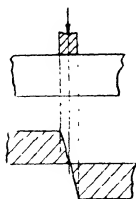
FIG. 107
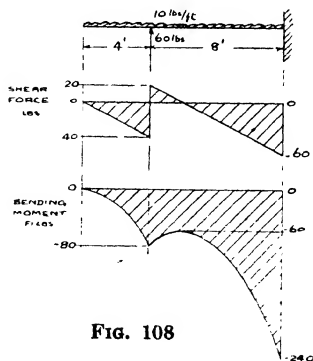
FIG. 108

EXAMPLE.—Draw the shearing force diagrams for the cantilever in Fig. 108, and use it to determine (*a*) the points where the maximum B.M. may occur, (*b*) to draw the Bending Moment diagram.

From the shearing force diagram it is seen that the shearing force passes through zero value at the 60 lb. load, at 6 feet from the fixed end, and at the fixed end. The maximum B.M. may occur at either of these points.

The following table shows how the bending moment diagram is obtained—

Position, Distance from Free End.	Area of S.F.D. between Points taken.	B.M., ft. lb.
0	0	0
1 ft.	-5	$0 - 5 = -5$
2 ft.	-15	$-5 -15 = -20$
3 ft.	-25	$-20 -25 = -45$
4 ft.	-35	$-45 -35 = -80$
5 ft.	15	$-80 +15 = -65$
6 ft.	5	$-65 + 5 = -60$
7 ft.	-5	$-60 - 5 = -65$
8 ft.	-15	$-65 -15 = -80$
9 ft.	-25	$-80 -25 = -105$
10 ft.	-35	$-105 -35 = -140$
11 ft.	-45	$-140 -45 = -185$
12 ft.	-55	$-185 -55 = -240$

It will be noticed that the point 6 feet from the fixed end is a point of minimum and not maximum bending moment, but as the slope is zero this is not apparent from the shearing force diagram until the bending moment diagram is drawn.
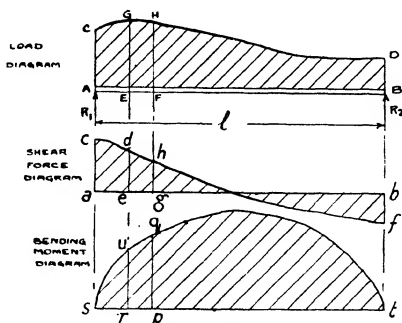
FIG. 109

Varying Distributed Loads.

The relation between B.M. and S.F. is very useful in drawing the bending moment diagram for a beam having a varying distributed load.

Let AB (Fig. 109) be a beam loaded with a varying distributed load, and let the shaded area $ABCD$ represent the load; such that the height of the line CD above AB at any point represents to some suitable scale the loading per unit length at that point.

If the height of the load diagram *ABDC* is to a scale of 1 unit represents x lb.ft., and the length to a scale of 1 unit represents y feet, the load on any length *EF* will equal the area $GHFE \times x \times y$.

To find the reactions, divide the beam up into a number of small lengths, find the load on each of these, and the distance of the centre of these lengths from some suitable point such as *A*. Then the algebraic sum of the moments of all these loads and the reactions about that point equals zero. If the position of the centroid of *ABDC* is known, the reactions may be found simply by taking moments about *A*. Then

$$WX - R_2 l = 0$$

$$R_1 + R_2 - W = 0,$$

where W = total load,

X = horizontal distance of the centroid of the load diagram or C.G. of the load from *A*.

To draw the shearing force diagram, let *ab* be the line of zero shearing force. At *A* the shearing force equals R_1; mark off *ac* to represent R_1. At any point *E* the shearing force will equal R_1 minus the load represented by the area of the load diagram to the left of *E*—*i.e.*, $R_1 - CGEA \times x \times y$; this is represented by *de* on the diagram. By taking several points in this way the complete shearing force diagram is obtained.

At any other point *F* the S.F. is equal to the S.F. at *E* minus the force represented by the area of the load diagram between *E* and *F*—*i.e.*, $de - GHFE \times x \times y$.

To obtain the bending moment diagram, let *st* be the line of zero bending moment. At *A* the B.M. is zero; at any point *E* the B.M. is represented by the area of the shearing force diagram to the left of this point—*i.e.*, B.M. at E = area $cdea \times z \times y$, where the scale of S.F. is 1 unit represents z lb. This is represented by *ur* in the bending moment diagram. At any other point *F* the B.M. is equal to the B.M. at *E*, plus the B.M. represented by the area of the shearing force diagram between *E* and *F*—*i.e.*, B.M. at $F = -ur + $ area $dhge \times z \times y$; this is represented by *pq* in the diagram. By taking several points in this way the complete bending moment diagram may be drawn.

EXAMPLE 1.—A beam 9 ft. long is freely supported at the ends, and carries a distributed load varying from 180 lb./ft. at one end to 0 lb./ft. at the other end. Draw the Shearing Force and Bending Moment diagrams.

The load diagram is shown in Fig. 110. This being a triangle, the horizontal distance of the C.G. from *A* will be one-third the length of the beam—*i.e.*, 3 feet.

$$\text{Total load} = \frac{180 \times 9}{2}$$

$$= 810 \text{ lb.}$$

To find R_1 and R_2,

$$3 \times 810 - 9R_2 = 0$$

$$R_2 = \frac{3 \times 810}{9}$$

$$= 270 \text{ lb.}$$

$$R_1 = 810 - 270$$
$$= 540 \text{ lb.}$$

Divide the beam up into nine 1 foot lengths, and find the loads on these lengths from the load diagram. From these loads the shearing force may be found at different points. Draw the shearing force diagram and find its area on each 1 foot length, which represents the difference in bending moment between each end of the 1 foot length considered. Find the bending moments at different points in this way and draw the bending moment diagram.

The shearing force and bending moment diagrams are shown in Fig. 110, and the calculations are given in the table (p. 117).
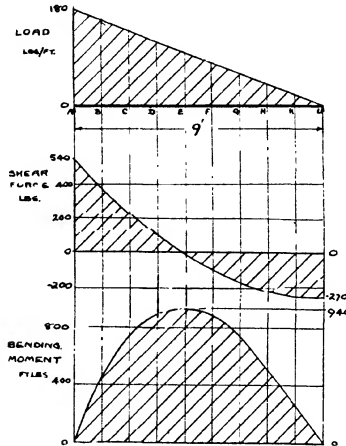
FIG. 110

EXAMPLE 2.—An 8 ft. wing rib is supported by two spars, one at 1 ft. from the leading edge and the other at 4 ft. 6 in. from the leading edge. If the rib is loaded in the manner shown by the load diagram in Fig. 111, draw the shearing force and bending moment diagrams.

Divide the beam up into 6-inch lengths, find the load on each length and its distance from C. Then by taking moments about C, find R_2.

Position.	Load represented by Area of Load Diagram.	Position	S.F.	Position.	Difference in B.M. represented by Area of S.F.D.	Position.	B.M.
	Lb.		Lb.		Ft./lb.		Ft./lb.
A—B	170	A	R_1 = 540	A—B	455	A	
B—C	150	B	540—170=370	B—C	295	B	455
C—D	130	C	370—150=220	C—D	155	C	455+295=750
D—E	110	D	220—130= 90	D—E	35	D	750+155=905
E—F	90	E	90—110= —20	E—F	— 65	E	905+ 35=940
F—G	70	F	— 20— 90= —110	F—G	—145	F	940— 65=875
G—H	50	G	—110— 70= —180	G—H	—210	G	875—145=730
H—K	30	H	—180— 50= —230	H—K	—250	H	730—210=520
K—L	10	K	—230— 30= —260	K—L	—270	K	520—250=270
		L	—260— 10= —270			L	270—270=0
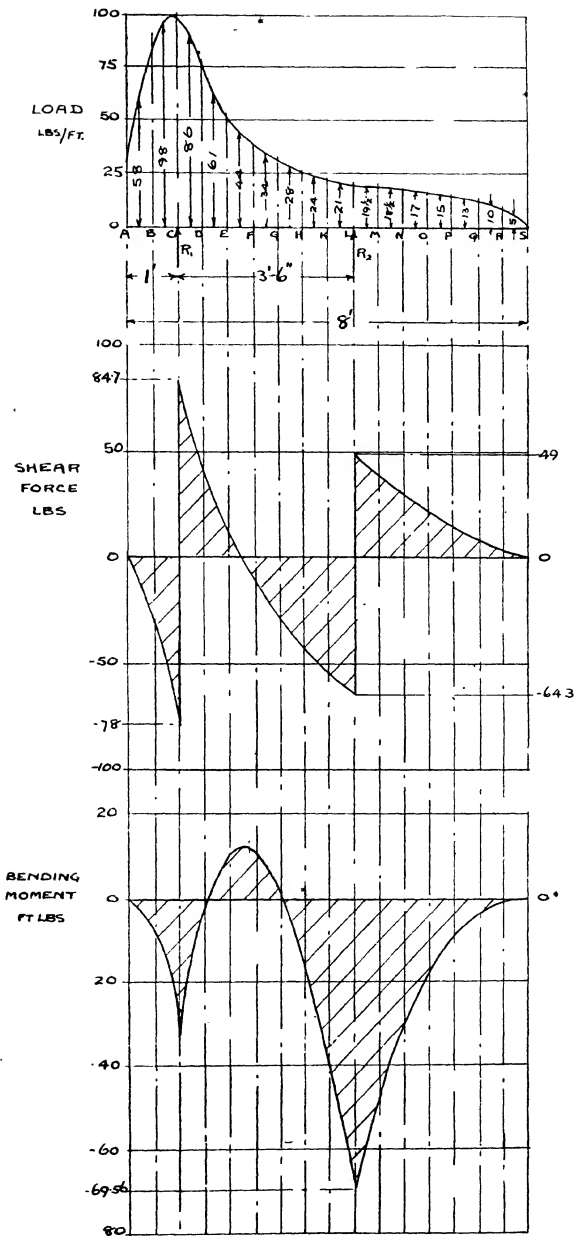
FIG. 111

Position.	Loads from Area of Load Diagram.	Distance from C.	Moments of Loads about C.
	Lb.	Ft.	Ft. lb.
A—B	29	0·75	−21·75
B—C	49	0·25	−12·25
C—D	43	0·25	+10·75
D—E	30·5	0·75	+22·9
E—F	22	1·25	+27·5
F—G	17	1·75	+29·75
G—H	14	2·25	+31·5
H—K	12	2·75	+33·0
K—L	10·5	3·25	+34·15
L—M	9·75	3·75	+36·55
M—N	9·25	4·25	+39·3
N—O	8·5	4·75	+40·4
O—P	7·5	5·25	+39·35
P—Q	6·5	5·75	+37·35
Q—R	5	6·25	+31·25
R—S	2·5	6·75	+16·9

Total load = 276 lb.   Total moment = 396·65 ft. lb.

From the above table, it is seen that

$$396 \cdot 56 - 3 \cdot 5 \, R_2 = 0$$

$$R_2 = \frac{396 \cdot 65}{3 \cdot 5}$$

$$= 113 \cdot 3 \text{ lb.}$$

$$276 - 113 \cdot 3 - R_1 = 0$$

$$R_1 = 162 \cdot 7 \text{ lb.}$$

The shearing force and bending moment diagrams are obtained in the same manner as in the previous example. The diagrams are shown in Fig. 111, and the calculations in the following table.

Position.	Loads from Area of Load Diagram.	Position.	S.F.	Position.	Difference in B.M. represented by Area of S.F.D.	Position.	B.M.
	Lb.		Lb.		Ft./lb.		Ft./lb.
A—B	29.	A	0	A—B	−7·25	A	0
B—C	49	B	−29	B—C	−26·75	B	−7·25
C—D	43	C	−29−49 = −78 and 162·7−78 = 84·7	C—D	31·6	C	−7·25−26·75 = −34
D—E	30·5	D	84·7−43 = 41·7	D—E	13·22	D	−34+31·6 = −2·4
E—F	22	E	41·7−30·5 = 11·2	E—F	0·1	E	−2·4+13·22 = 10·82
F—G	17	F	11·2−22 = −10·8	F—G	−9·65	F	10·82+0·1 = 10·92
G—H	14	G	−10·8−17 = −27·8	G—H	−17·4	G	10·92−9·65 = 1·27
H—K	12	H	−27·8−14 = −41·8	H—K	−23·9	H	1·27−17·4 = −16·13
K—L	10·5	K	−41·8−12 = −53·8	K—L	−29·53	K	−16·13−23·9 = −40·03
L—M	9·75	L	−53·8−10·5 = −64·3 and 113·3−64·3 = 49	L—M	22·06	L	−40·03−29·53 = −69·56
M—N	9·25	M	49−9·75 = 39·25	M—N	17·31	M	−69·56+22·06 = −47·5
N—O	8·5	N	39·25−9·22 = 30	N—O	12·86	N	−47·5 +17·31 = −30·19
O—P	7·5	O	30−8·5 = 21·5	O—P	8·86	O	−30·19+12·86 = −17·33
P—Q	6·5	P	21·5−7·5 = 14	P—Q	5·36	P	−17·33+ 8·86 = −8·47
Q—R	5	Q	14−6·5 = 7·5	Q—R	2·5	Q	−8·47+5·36 = −3·11
R—S	2·5	R	7·5−5 = 2·5	R—S	0·62	R	−3·11+2·5 = −0·61
		S	2·5−2·5 = 0			S	−0·61+0·62 = 0

CHAPTER XI

STRESSES IN BEAMS

In dealing with simple bending, the following assumptions are made—

(1) A plane cross-section at right angles to the plane of bending always remains in a plane.

(2) The Modulus of Elasticity of the material is the same for tension and compression.

(3) The material is free to expand or contract longitudinally and laterally as if it were made of separate layers.

(4) The material obeys Hooke's Law—*i.e.*, the stress is proportional to the strain.

It is important that these assumptions should be known, as in some cases, such as reinforced concrete beams, they do not hold good. They are, however, sufficiently accurate to be applied to all beams used in aircraft at the present time.

Neutral Axis.

Referring to Fig. 112, let *BD* and *CE* be two parallel cross-sections of a beam, distance *l* apart. On bending they will no longer be parallel, *BC* having increased in length, and *DE* decreased. There must be a plane where the mater al remains the same ; this plane *NP* is called
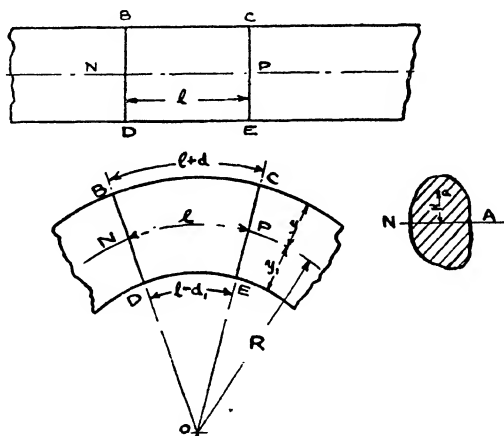
Fig. 112

the Neutral Plane, and its line of intersection with the cross-section of the beam is called the Neutral Axis (*NA*). This neutral axis passes through the centroid of the section, as will be explained later.

Stresses due to Bending.

Let f = the maximum stress in the material at the section considered,

   y = the distance of the further outside of the section from the N.A.

   M = the bending moment of the section,

   I = the moment of inertia of the area of the section about the N.A.,

   E = modulus of elasticity of the material,

   R = radius of curvature of the beam at the N.A.

Consider the length of l on the neutral plane of the bent beam. At a distance y outwards it has increased to $l+d$, and at a distance y_1 inwards it has decreased to $l-d_1$.

If the section is symmetrical about N.A.,

$$y = y_1 \text{ and } d = d_1$$

$$\frac{BC}{CO} = \frac{NP}{PO}$$

$$i.e., \quad \frac{l+d}{R+y} = \frac{l}{R}$$

$$Rl+Rd = Rl+yl$$

$$Rd = yl$$

$$\frac{d}{l} = \frac{y}{R}$$

But $\dfrac{d}{l} = \dfrac{\text{Max. increase in length}}{\text{original length}}$ = max. strain,

$\dfrac{\text{Stress}}{\text{Strain}} = E$, or max. strain $= \dfrac{f}{E}$

$$\therefore \quad \frac{f}{E} = \frac{y}{R} \text{ or } \frac{f}{y} = \frac{E}{R}$$

Now consider a small element of area a, distance x from N.A., subjected to a stress of f_1.

The total force in this area $= f_1 a \left(\text{stress} = \dfrac{\text{load}}{\text{area}}\right)$ and its moment about N.A. $= f_1 a x$.

The total moment about N.A. = Moment of Resistance = the sum of the moments of all the elements of area which make up the whole cross-section $= \Sigma f_1 a x$.

Moment of Resistance $= M$,

$$\therefore \quad M = \Sigma f_1 a x.$$

If assumption (1) holds good, the variation of strain must be uniform across the section ; for if this were not so the plane sections BD and CE (Fig. 112) would on bending become distorted and no longer remain in a plane.

We may therefore represent the variation of strain by the diagram in Fig. 113A.

The stress being proportional to the strain for tension and compression, this diagram will also show the stress distribution.

If *FG* represents f, then *KH* will represent f_1.

From similar triangles,

$$\frac{f}{y} = \frac{f_1}{x}$$

$$\therefore f_1 = \frac{fx}{y}$$

From above, $M = \Sigma f_1 ax.$
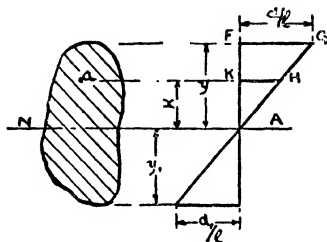
FIG. 113A

Substituting for $_1$

$$M = \frac{f}{y} \Sigma ax^2$$

But $\Sigma ax^2 = I,$

$$\therefore M = \frac{f}{y} I$$

$$\text{or } \frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

This is an important relation from which we get $M = \dfrac{f}{y} I$, sometimes written $M = fZ$, where Z is a constant depending on the shape and size of the section, called the Modulus of Section, and equal to $\dfrac{I}{y}$

Values of I, y and Z for various standard sections are given in Appendix I.

Methods of finding I for both thick and thin sections are given in Volume II of this series.

Position of Neutral Axis.

Fig. 113B shows the section of a beam and the stress distribution across it.

By proportion : $\dfrac{f}{y} = \dfrac{f_1}{z_1} = \dfrac{f_2}{z_2}$

For equilibrium—

Compressive reaction = Tensile reaction
$$\Sigma f_1 t_1 x_1 = \Sigma f_2 t_2 x_2$$

Substituting for f_1 and f_2

$$\Sigma \frac{f}{y} z_1 t_1 x_1 \;=\; \Sigma \frac{f}{y} z_2 t_2 x_2$$

$$\therefore \; \Sigma z_1 t_1 x_1 \;=\; \Sigma z_2 t_2 x_2$$

i.e., Moment of area above N.A.=Moment of area below N.A.

Therefore N.A. passes through the centroid of section.
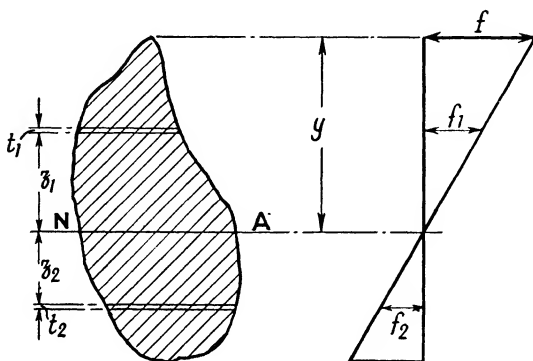
Fig. 113B

The Effect of the Shape of the Section.

Distribution of Material for a Given Depth.—In Fig. 114 the strength of an " I " section beam is compared with that of a rectangular one of equal area, weight and depth, and the same material.

The maximum permissible stress, and therefore the stress distribution, will be the same in each case.

The internal force or load distributions over the sections are found by dividing the sections into small strips and finding the products of their respective areas and stresses.

In this case the width of each strip is taken as 0·1 inch.

The area of each strip on the rectangle is therefore 0·1 sq. inch.

Load/Strip = Stress × Area of strip.

This decreases uniformly from 1 ton tension at the top to zero at the centre and 1 ton compression at the bottom.

In the " I " section the areas will be 0·2 sq. inch in the flanges and 0·05 sq. inch in the web. This gives the load distributions shown on the right.

The total loads in each case are the product of the average load/strip and the number of strips, and may be assumed to act through the centroid of the load graph.

Thus the rectangular beam has a resistance of 7·5 tons tension and compression, acting at 2 inches from each other, and the moment of resistance will equal $2 \times 7\cdot 5 = 15$ in. tons.

Whilst the " I " beam gives resistances of 10 tons at $2\frac{1}{3}$ inches, and its moment of resistance will equal $2\frac{1}{3} \times 10 = 23\frac{1}{3}$ in. tons.

i.e., The " I " section will withstand $8\frac{1}{3}$ in. tons extra bending moment.

The above should make it clear that by putting the bulk of the

LOAD DISTRIBUTION

BEAM SECTIONS

STRESS DISTRIBUTION

RECTANGLE

I SECTION
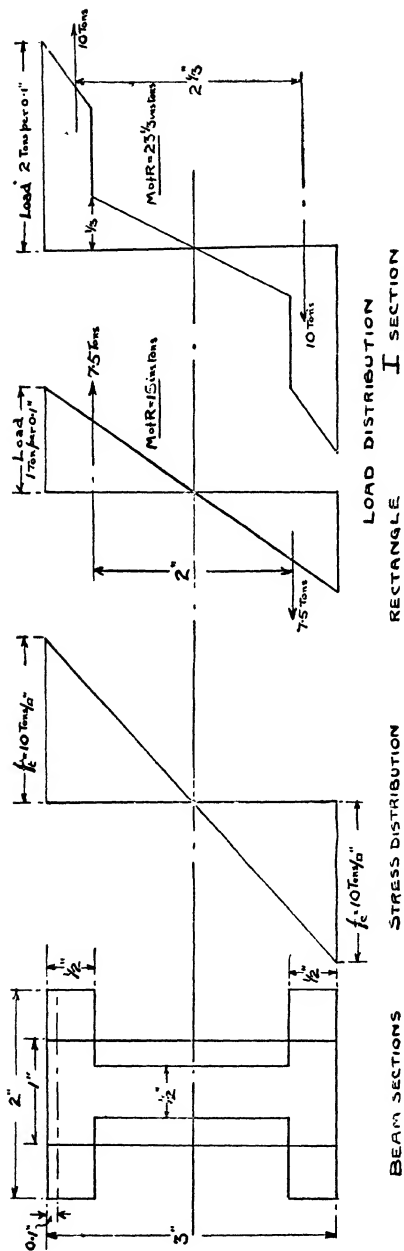
FIG. 114

material at a distance from the neutral axis where the stress is a maximum—

    (*a*) The total resisting force is increased ;

    (*b*) The distance of its line of action is increased ;

    (*c*) The moment of resistance is increased ;

    (*d*) The strength/weight ratio is increased.

Effect of Depth.

In Fig. 115 the strength of two rectangular beams of equal area and weight, and the same material, but of different depth, is compared in a similar manner as the previous case.

In the case of the 6 inches deep beam the area of each 0·1 inch strip is 0·05 sq. inches, and the Load/Strip will decrease uniformly from
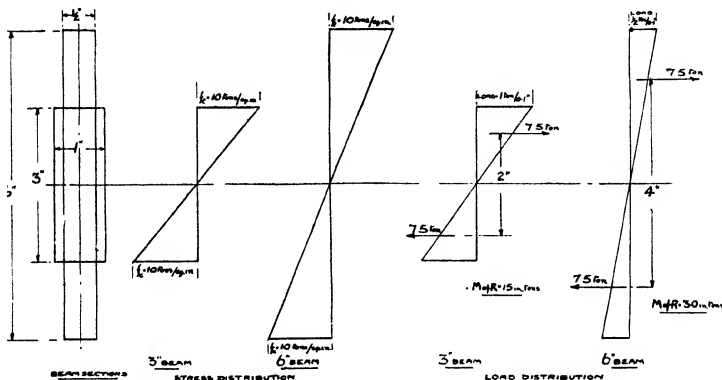
FIG. 115

½ ton tension at the top to zero at the centre and ½ ton compression at the bottom. This gives a resistance of 7·5 tons tension and compression as for the 3 inches deep beam, but they act 4 inches from each other, thus giving a moment of resistance of 30 in. tons. *i.e.*, By doubling the depth of the beam and keeping the area the same, the beam is able to withstand twice the bending moment.

It follows that by increasing the depth of a beam—

    (*a*) The total resisting force is unaltered ;

    (*b*) The distance of its line of action is increased ;

    (*c*) The moment of resistance is increased ;

    (*d*) The strength/weight ratio is increased.

Unsymmetrical Sections.

In Fig. 116A the strength of a symmetrical rectangular beam is compared with a " T " section beam—that is, unsymmetrical about the neutral axis. Both beams are of the same cross-sectional area and weight, and the same material.

In the case of the " T " section, the N.A., which is through the centroid, will be towards the top. As this is the point of zero stress, and as the stress varies uniformly across the section, the stress at the

top will be less than the stress at the bottom. In the case considered
the N.A. is 1 inch from the top, and, taking the maximum permissible
stress as 10 tons/sq. inch, this gives a stress of 5 tons/sq. inch at the
top.

The load distribution is found as for the previous cases ; this shows
a resisting force of 5 tons for the " T " beam, compared to 7·5 tons
for the rectangular beam, and a moment of resistance of 10 in. tons
and 15 in. tons for the " T " and rectangular beams respectively.
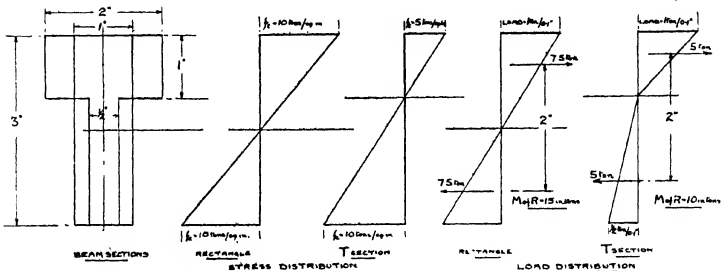
FIG. 116A

It may be seen that with an unsymmetrical section—

    (a) The maximum stress is reduced on the outside nearest the
        N.A.

    (b) The total resisting force is decreased.

    (c) The moment of resistance is decreased.

    (d) The strength/weight ratio is decreased.

Elastic Stability.

In dealing with beams and compression members made of very thin
metal, it may happen that the member will fail in compression before
the elastic limit of the material is reached, and is said to be elastically
unstable. Thus a thin metal spar subjected to pure bending is elas-
tically unstable if it should fail when $\dfrac{M}{Z}$ is less than the compressive
proof stress of the material. This takes place due to the formation of
a " wave " or local buckle, and may be overcome by stiffening the
member by means of corrugations. The Frontispiece shows a spar
flange failing by elastic instability. A fuller explanation of this
phenomenon is given in Chapter XIII.

Shear Stress.

It has been shown in Chapter V that a shear stress cannot exist in
a body without an equal shear stress at right-angles. Thus the forces
in a beam will not only produce a transverse shear stress, but also a
longitudinal shear stress of equal intensity. This stress, which varies
across the length and depth of the beam, may be found in the following
manner—

In Fig. 116B let AB be a cross-section of a beam, distance dl from

another cross-section, such that dl is very small. At any height x above N.A. let the thickness of the section be t_1.
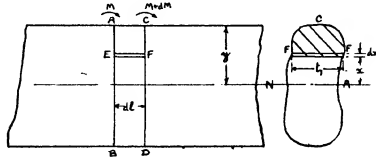
FIG. 116B

Let the bending moment at $AB = M$, and at $CD = M + dM$.

The bending stress on AB at any height x above NA,

$$f_1 = \frac{Mx}{I} \qquad \left(\frac{M}{I} = \frac{f}{y} = \frac{f_1}{x} \right)$$

On the element of area $t_1 . dx$ the longitudinal force at AB = stress × area $= f_1 t_1 . dx$.

Substituting for $f_1 = \dfrac{M . x . t_1 . dx}{I}$

The bending stress on section CD at any height x above N.A.

$$f_2 = \frac{(M + dM)x}{I}$$

and the longitudinal force on the area $t_1 dx$ at CD

$$= f_2 t_1 \, dx$$
$$= \frac{(M + dM) \, x \, t_1 \, dx}{I}$$

The extra longitudinal force on the area $t_1 . dx$ at CD over that at AB equals the difference of the longitudinal forces on $t_1 . dx$ at CD and AB

$$= \frac{(M + dM) \, x \, t_1 \, dx}{I} - \frac{M \, x \, t_1 dx}{I}$$
$$= \frac{dM \, x \, t_1 \, dx}{I}$$
$$= \frac{dM}{I} \times \text{Moment of element of area } t_1 \, dx \text{ about N.A.}$$

The total excess force on the area FCF above $F - F$ at CF over that at AE equals the sum of the excess forces on all the elements of area $t_1 \, dx$, making up the whole area FCF

$$= \frac{dM}{I} \times \text{Moment of area } FCF \text{ about N.A.}$$

As the resultant horizontal force on the portion $ACFE$ must be zero from conditions of equilibrium, the excess force at CF must be balanced by a horizontal shearing force on the surface EF.

If f_{s1} is the longitudinal shearing stress at a distance x from N.A., the longitudinal shearing force

$$= \text{shear stress} \times \text{area}$$

$$= f_{s1}\, t_1\, dl = \frac{dM}{I} \times \text{Moment of area } FCF \text{ about N.A.}$$

$$\text{or } f_{s1} = \frac{dM}{dl \cdot I \cdot t_1} \times \text{Moment of area } FCF \text{ about N.A.}$$

The shearing force on any cross-section of a beam—

$$F = \frac{dM}{dl} \quad \text{(See Chapter X, Relations between B.M. and S.F.)}$$

$$\therefore f_{s1} = \frac{F}{It_1} \times \text{Moment of area } FCF \text{ about N.A.}$$

$$= \frac{F}{It_1} \times \text{Area } FCF \times \text{distance of centroid of F.C.F. from N.A.}$$

This longitudinal shear stress is equal to the transverse shear stress, and for all practical purposes is greatest at the neutral axis (N.A.).

Shear stress at N.A. for any section of a beam

$$f_s = \frac{FA_1 z}{It}$$

where F = shearing force at section considered,

      I = moment of inertia of section about N.A.,

      A_1 = area of part section above N.A.,

      z = distance of centroid of part section A_1 above N.A.,

      t = total thickness of beam at N.A.

Wagner Beam.

It often happens that only a very thin web is required to take the shear forces in a beam, and as in aeronautical engineering we cannot afford to waste weight, a thin web is often used. When such a beam is made up of two flanges joined by a deep thin plate web, as illustrated in Fig. 117B, it is called a Wagner beam. Such a beam cannot be treated for shear by the method given above.

It was explained in Chapter V that whenever a shear stress is present in a body the material is also subjected to tensile and compressive stresses of equal intensity at 45° to the shear. When the web is thick these tensile and compressive stresses are withstood by the material, and the web will only fail by shear. When, however, the web is thin it will be elastically unstable under compression, and this will cause it to act in a similar manner to a braced girder.

The braced girder, shown subjected to a load P in Fig. 117A, has compression and tension in its flanges due to bending. The shear force in the first panel is resisted by tension in AC and by compression in BD. If, however, BD is a wire it will bow, taking practically no load. The structure will not collapse as AC keeps the frame rigid. Now consider the cross-bracing replaced by a thin plate web, and we have the Wagner beam, Fig. 117B. Shear now puts the web in tension across AC, and compression across BD. BD is elastically unstable, so the shear is resisted by tension across AC. The web will form waves as it cannot form a single bow, as did the bracing wire, due to

the restriction of the flanges. These waves, illustrated in the Frontispiece, are not a sign of failure ; they are elastic and disappear when the load is removed, as did the bow in the bracing of the girder.

Stresses in Wagner Beams.

Consider the Wagner beam, Fig. 117c, to be divided in two by the section X-X. The portion of beam to the right of X-X is in equilibrium under the applied force P, the tension in the top flange F_t, the compression in the bottom flange F_c and the tension in the web F_w.

$$F_w = P \sin 45° = P\sqrt{2}$$

Length of web AB on which it acts $= \dfrac{d}{\sqrt{2}}$

Taking moments about the bottom flange at X-X—

$$Px - F_t d - \frac{F_w d}{2\sqrt{2}} = 0$$

$$F_t = \frac{Px - \dfrac{F_w d}{2\sqrt{2}}}{d} = \frac{Px}{d} - \frac{F_w}{2\sqrt{2}}$$

$$= \frac{Px}{d} - \frac{P\sqrt{2}}{2\sqrt{2}} = \frac{Px}{d} - \frac{P}{2}$$

$$i.e., F_t = \frac{M}{d} - \frac{S}{2}$$

where M is the bending moment at the section, and S is the shearing force.

In the same way—

$$F_c = \frac{M}{d} - \frac{S}{2}$$

Stress in web,

$$f_w = \frac{F_w}{\text{Area } A\text{-}B} = \frac{F_w}{td \sin 45°} \text{ (where } t = \text{thickness)}$$

$$= \frac{P\sqrt{2}}{\dfrac{td}{\sqrt{2}}} = \frac{2P}{td} = \frac{2S}{td}$$

Tensile stress in flange,

$$f_t = \frac{F_t}{A} \text{ (where } A \text{ is the cross-sectional area of flange)}$$

The compressive stress in the flange will depend on its rigidity as a strut. If there are no stiffeners the flange will bow inwards, due to the pull of the webs, as shown in Fig. 117d. To prevent this stiffeners are fitted, as shown in Fig. 117b, also the end member must be stiff.

EXAMPLE 1.—A 10 ft. beam is freely supported at each end, and carries a load of 500 lb. evenly distributed over its whole length. Find the minimum width, if it is of rectangular cross-section and 2 inches deep The stress is not to exceed 4 tons/sq. inch.
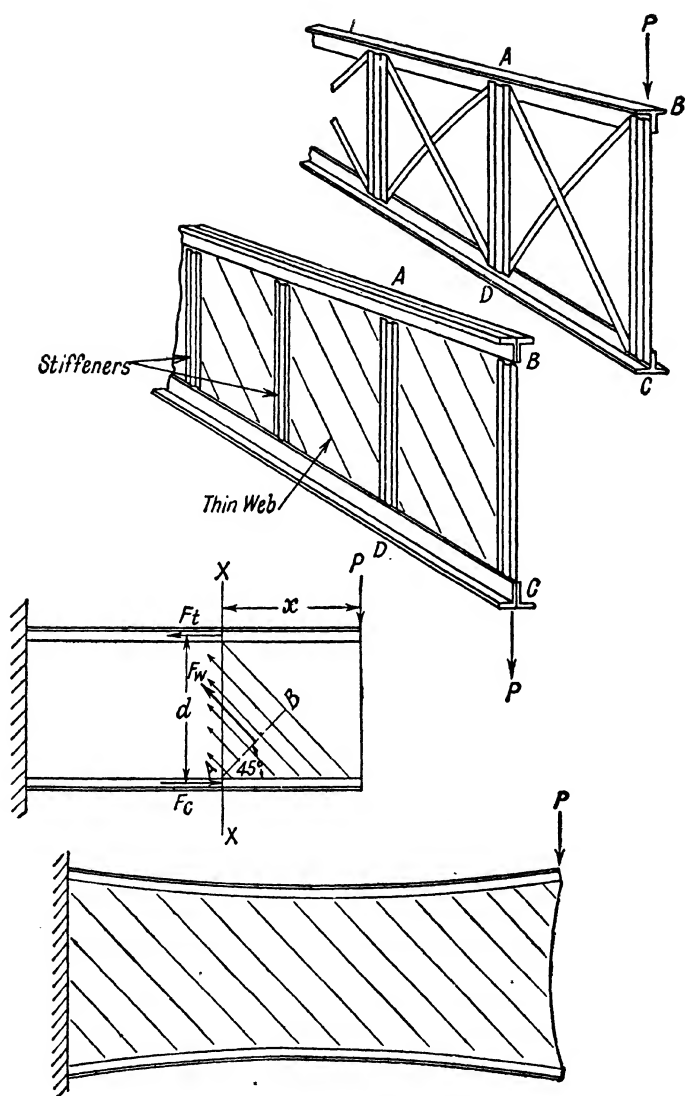
FIG. 117 A, B, C, D

The maximum B.M. occurs at the centre.

Maximum B.M. $= 5 \times 250 - 2 \cdot 5 \times 250$

$\quad\quad\quad\quad\quad = 1250 - 625$

$\quad\quad\quad\quad\quad = 625$ ft. lb.

$$M = fZ$$

$$= \frac{fbd^2}{6}$$

$$b = \frac{6M}{fd^2}$$

$$= \frac{6 \times 625 \times 12}{4 \times 2240 \times 4}$$

$$= 1 \cdot 26 \text{ inches.}$$

EXAMPLE 2.—The following particulars apply to an aeroplane metal spar which is symmetrical about the neutral axis—

Moment of Inertia of section	$= 0 \cdot 3$ inch.[4].
Total area of section	$= 0 \cdot 25$ sq. inch.
Distance of centroid of half section above N.A.	$= 1 \cdot 1$ inch.
Thickness of web at N.A.	$= 0 \cdot 03$ inch.
Maximum allowable shear stress	$= 60$ tons/sq. inch.

Find the maximum shear force the spar will withstand.

$$f_s = \frac{FA_1 z}{It}$$

$$F = \frac{f_s It}{A_1 z}$$

$$= \frac{60 \times 0 \cdot 3 \times 0 \cdot 03}{\frac{0 \cdot 25}{2} \times 1 \cdot 1}$$

$$= 3 \cdot 93 \text{ tons.}$$

EXAMPLE 3.—Find the maximum stress in the web of a Wagner beam with a 10 in. deep and ·028 in. thick web, when subjected to a maximum shear force of 7 tons.

$$\text{Stress in web} = \frac{2S}{td}$$

$$= \frac{2 \times 7}{\cdot 028 \times 10} = 50 \text{ ton/sq. in.}$$

EXAMPLE 4.—Show by means of graphs the distribution of shear stress over the section of a beam where the shearing force is 10 tons.

(a) If of rectangular section 2 inches wide and 4 inches deep ;

(b) If of " I " section 3 inches wide, 4 inches deep, 1 inch thick flanges and 1 inch thick web.

$$f_{s1} = \frac{F}{It_1} \times \text{Area of section above the line chosen} \times \text{Distance of centroid of this section above N.A.}$$

$$I \text{ for rectangle} = \frac{bd^3}{12} = \frac{2 \times 4^3}{\cdot 12} = 10\tfrac{2}{3} \text{ inches.}^4$$

RECTANGULAR SECTION.

Distance above or below N.A.	t_1	Area.	Centroid above N.A.	f_{s1}
Inches.	Inches.	Sq. inches.	Inches.	Tons/sq. inch.
2	2	0	—	0
1½	2	1	1·75	0·82
1	2	2	1·5	1·41
½	2	3	1·25	1·76
0	2	4	1	1·875

$$I \text{ for “ I ” section } = \frac{bd^3 - b_1d_1^3}{12}$$

$$= \frac{3 \times 4^3 - 2 \times 2^3}{12}$$

$$= 14\tfrac{2}{3} \text{ inches.}[4]$$

“ I ” SECTION.

Distance above or below N.A.	t_1	Area.	Centroid above N.A.	f_{s1}
Inches.	Inches.	Sq. inches.	Inches.	Tons/sq. inch.
2	3	0	—	0
1½	3	1·5	1·75	0·60
1	3	3	1·5	1·02
1	1	3	1·5	3·07
½	1	3·5	1·39	3·32
0	1	4	1·25	3·41

The graphs are shown in Fig. 118.

It should be noted that the maximum shear stress occurs at the neutral axis.
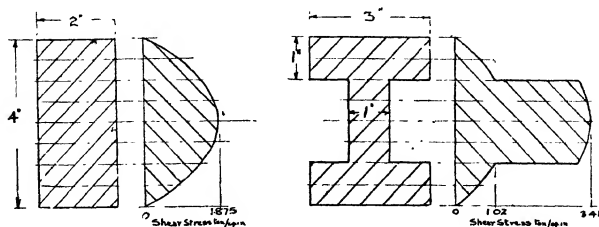
FIG. 118

EXAMPLE 5.—For the beam illustrated in Fig. 119, find the maximum stresses due to bending and shear. Neglect the weight of the beam.

Find the reactions by taking moments about A :

$$6 \times 1200 + 14 \times 800 - 20 \, R_1 = 0.$$

$$R_1 = \frac{7200 + 11200}{20}$$

$$= 920 \text{ lb.}$$

$$R_2 = 1200 + 800 - 920$$
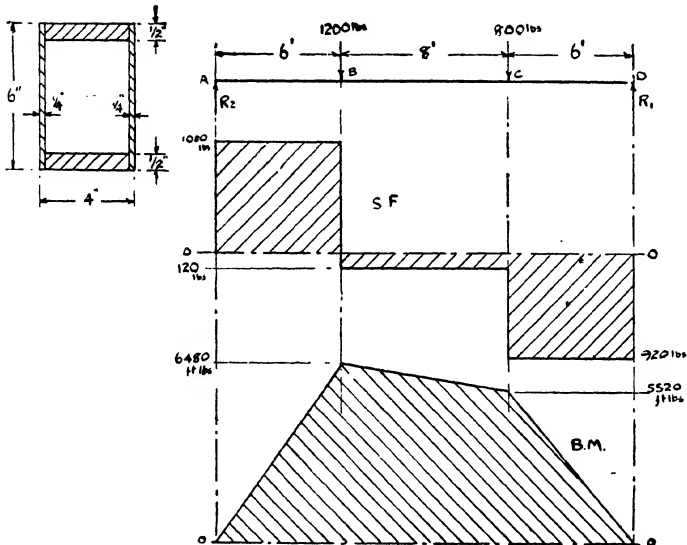
$$= 1,080 \text{ lb.}$$

FIG. 119

Draw the shearing force and bending moment diagrams, from which it will be seen that—

Max. shearing force   =   1,080 lb.

Max. bending moment   =   6,480 ft. lb.
                      =   77,760 in. lb.

$$\text{Max. shear stress} = \frac{F A_1 z}{It}$$

$$A_1 = \tfrac{1}{2} \times 4 + \tfrac{1}{2} \times 2 \cdot 5$$

$$= 2 + 1 \cdot 25$$

$$= 3 \cdot 25 \text{ sq. inches.}$$

To find z take moments of area about N.A.

$$\tfrac{1}{2} \times 4 \times 2 \cdot 75 + \tfrac{1}{2} \times 2 \cdot 5 \times 1 \cdot 25 - 3 \cdot 25\, z = 0.$$

$$z = \frac{5 \cdot 5 + 1 \cdot 563}{3 \cdot 25}$$

$$= 2 \cdot 17 \text{ inches.}$$

$$I = \frac{bd^3 - b_1 d_1^3}{12}$$

$$= \frac{4 \times 6^3 - 3 \cdot 5 \times 5^3}{12}$$

$$= 35 \cdot 54 \text{ inches}^4.$$

Max. shear stress

$$f_s = \frac{F A_1 z}{I t}$$

$$= \frac{1080 \times 3 \cdot 25 \times 2 \cdot 17}{35 \cdot 54 \times 0 \cdot 5}$$

$$= 428 \cdot 6 \text{ lb./sq. inch.}$$

Max. bending stress

$$f = \frac{My}{I}$$

$$= \frac{77760 \times 3}{35 \cdot 54}$$

$$= 6,560 \text{ lb./sq. inch.}$$

It should be noticed how small the stress due to shear is compared to the stress due to bending. For this reason shear stress in beams is usually neglected, unless the beam is very short when the shearing force will be large compared to the bending moment, or in the case of beams, with very thin webs. It must always be considered when dealing with thin metal spars.

DEFLECTION OF BEAMS

Consider the cantilever AB (Fig. 120) under any system of loading, and let δ=the deflection at the end of B, due to the distortion of a small elementary transverse layer of length dl, at a distance l_1 from the end B.

If e is the elongation of dl at a distance x above the neutral axis from similar triangles

$$\frac{\delta}{e} = \frac{l_1}{x}$$

$$\delta = \frac{el_1}{x}$$

Let f_1 = stress at distance x above N.A.
and M = bending moment at elementary strip.

Then $f_1 = \frac{Mx}{I}$

Strain at distance x above N.A.

$$= \frac{e}{dl} = \frac{f_1}{E}$$

$$e = \frac{dl \cdot f_1}{E}$$

Substituting for f_1

$$e = \frac{dl \cdot Mx}{EI}$$

Substituting for e in

Fig. 120

$$\delta = \frac{el_1}{x}$$

$$\delta = \frac{dl \cdot Mx \cdot l_1}{EIx}$$

$$= \frac{dl \cdot Ml_1}{EI}$$

But $dl \cdot Ml_1$ = Moment of elementary strip of bending moment diagram about B ;

$$\therefore \ \delta = \frac{1}{EI} \times \text{Moment of elementary strip of bending moment diagram about } B.$$

Total deflection at B =

$$D = \frac{1}{EI} \times \text{Sum of the moments of all the elementary strips making up the whole bending moment diagram.}$$

$$D = \frac{1}{EI} \times \text{Moment of whole bending moment diagram about } B,$$
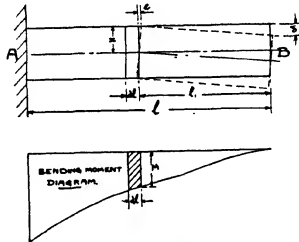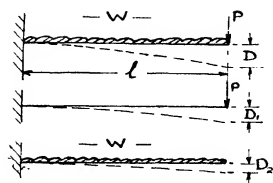
Fig. 124
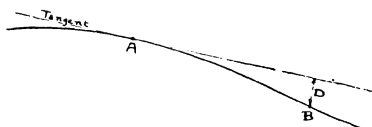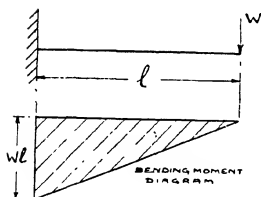
Fig. 121

Fig. 122
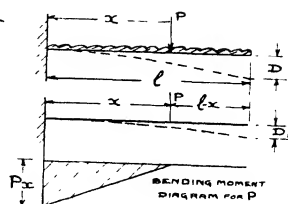
Fig. 125
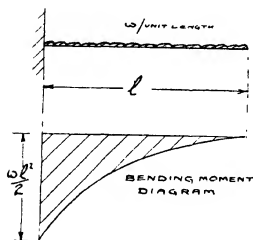
Fig. 123
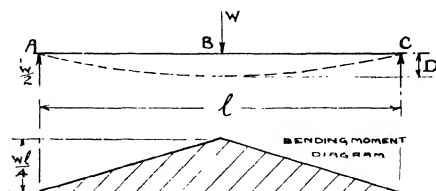
Fig. 126
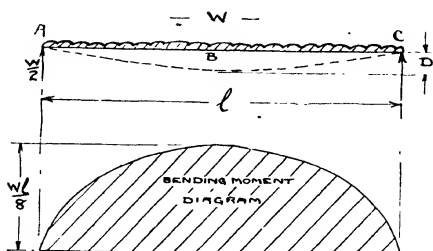
Fig. 127

or the deflection of a beam at a point B relative to a point A (Fig. 121) is equal to

$$\frac{1}{EI} \times \text{Moment of the bending moment diagram between } A \text{ and } B \text{ about } B, \text{ assuming } E \text{ and } I \text{ are constant.}$$

Change of slope due to straining of element

$$= \frac{\delta}{l_1} \text{radians}$$

$$= \frac{dl \cdot M}{EI} \text{ radians}$$

Total change of slope between A and B =

$$\theta = \frac{1}{EI} \, \Sigma \, dl \cdot M$$

$$= \frac{1}{EI} \times \text{Area of bending moment diagram between } A \text{ and } B. \text{ radians.}$$

Particular Cases.

Cantilevers.—A cantilever of length l, loaded at the free end with a concentrated load W. The beam and bending moment diagram is illustrated in Fig. 122.

Maximum deflection—*i.e.*, at the free end

$$D = \frac{1}{EI} \times \text{Moment of B.M. diagram about free end}$$

$$= \frac{1}{EI} \times \frac{Wl \times l}{2} \times \frac{2l}{3}$$

$$= \frac{Wl^3}{3EI}$$

$$\text{and slope} = \frac{Wl^2}{2EI}$$

Cantilever of length l, loaded with a total load W evenly distributed over the length with an intensity of w per unit length. Referring to Fig. 123, the curve of bending moment is a parabola, and the area of the diagram is $\frac{1}{3}$ maximum height $\times$ length. The centroid is $\frac{3}{4}$ length from the zero end.

$$\text{Deflection} = \frac{1}{EI} \times \frac{1}{3} \, \frac{wl^2}{2} \times l \times \tfrac{3}{4}l$$

$$= \frac{wl^4}{8EI} = \frac{Wl^3}{8EI}$$

Cantilever with a concentrated and distributed load.

Let the concentrated load be at the free end and equal to P, and W be a load evenly distributed over the whole length.

Referring to Fig. 124, the total deflection D is equal to the deflection D_1 caused by the load P plus the deflection D_2 caused by the distributed load W.

$$D_1 = \frac{Pl^3}{3EI}$$

$$D_2 = \frac{Wl^3}{8EI}$$

$$D = D_1 + D_2 = \frac{Pl^3}{3EI} + \frac{Wl^5}{8EI}$$

$$= \frac{l^3}{EI}\left(\frac{P}{3} + \frac{W}{8}\right)$$

Now consider that the concentrated load is a distance x from the fixed end (Fig. 125).

$$D = D_1 + D_2$$

$$D_2 = \frac{Wl^3}{8EI} \text{ (as before).}$$

$$D_1 = \frac{1}{EI} \times \text{ Moment of B.M. diagram of } P \text{ about free end.}$$

$$= \frac{1}{EI} \times \frac{Px \times x}{2} \times \left(\frac{2x}{3} + l - x\right)$$

$$= \frac{Px^2}{2EI}\left(l - \frac{x}{3}\right)$$

$$D = \frac{Px^2}{2EI}\left(l - \frac{x}{3}\right) + \frac{Wl^3}{8EI}$$

$$= \frac{1}{2EI}\left[Px^2\left(l - \frac{x}{3}\right) + \frac{Wl^3}{4}\right]$$

Beams Supported at the Ends.

A beam of length l supported at each end and carrying a concentrated load W at the centre (Fig. 126).

Each reaction $= \dfrac{W}{2}$

The deflection is a maximum at the centre B, but it is easier to assume B as fixed and the ends A and C deflected upwards.

Then deflection D

$$= \frac{1}{EI} \times \text{ Moment of bending moment diagram between } B \text{ and } C, \text{ about } C.$$

$$= \frac{1}{EI} \times \tfrac{1}{2}\frac{Wl}{4} \times \frac{l}{2} \times \frac{2}{3} \times \frac{l}{2}$$

$$= \frac{Wl^3}{48\,EI}$$

$$\text{Slope at ends} = \tfrac{1}{2}\frac{Wl}{4} \times \frac{l}{2} \times \frac{1}{EI}$$

$$= \frac{Wl^2}{16EI}$$

A beam of length l supported at the ends and carrying a total load W evenly distributed over the whole length, with an intensity of w per unit length (Fig. 127).

Each reaction $= \dfrac{W}{2}$

As in the previous case,

deflection $D = \dfrac{1}{EI} \times$ Moment of bending moment diagram between B and C, about C,

The curve of bending moment is a parabola, and the area of the diagram is $\frac{2}{3}$ maximum height × length. The centroid of the half between B and C is $\frac{5}{8} \dfrac{l}{2}$ from G.

$$D = \frac{1}{EI} \times \frac{2}{3} \frac{Wl}{8} \times \frac{l}{2} \times \frac{5}{8} \frac{l}{2}$$

$$= \frac{5Wl^3}{384EI} = \frac{5wl^4}{384EI}$$

A beam of length l supported at the ends and carrying a load W, uniformly distributed over the length and a concentrated load P at the centre.

The total deflection will equal the deflection due to the distributed load W, plus the deflection due to the concentrated load P.

Deflection due to $W = \dfrac{5Wl^3}{384EI}$

Deflection due to $P = \dfrac{Pl^3}{48EI}$

Total deflection $= \dfrac{5Wl^3}{384EI} + \dfrac{Pl^3}{48EI}$

$\qquad\qquad\qquad = \dfrac{l^3}{48EI} \ (\tfrac{5}{8}W + P).$

Examples of unsymmetrically loaded beams are better solved by the use of calculus, owing to the difficulty of determining the point of maximum deflection and centroid of the bending moment diagram. They are therefore not included in this book, but are left for when the student has studied further mathematics, and can read the more advanced works, to which this forms an introduction.

General Conclusions.

It will have been noticed from the foregoing that for a given type of loading the maximum deflection is
    (a) Proportional to the total load ;
    (b) Proportional to the cube of the length ;
    (c) Inversely proportional to the modulus of elasticity of the material ;
    (d) Inversely proportional to the moment of inertia of the section.

Note it is not dependent on the strength of the material, so long, of course, that it is within the elastic limit. It is this fact that makes it important to consider the elasticity as well as the strength when choosing a material for beams, such as aeroplane spars, which have longitudinal as well as transverse loads. The stress in a longitudinally loaded beam greatly increases with the deflection.

Effect of End Load.

A load along the longitudinal axis of a beam is usually termed an End Load.

The total stress in such a beam will be equal to the stress due to bending plus the direct stress due to end load.

The stress due to bending $= \dfrac{My}{I} = \dfrac{M}{Z}$

The stress due to end load $= \dfrac{P}{A}$

where $P =$ end load, and $A =$ area of cross-section.

The end load has the effect of increasing or decreasing the bending moment M, depending on whether it is compressive or tensile.

Referring to Fig. 128, case (a) shows a beam simply loaded at the centre; the maximum bending moment will be at the centre and equal to $\dfrac{Wl}{4}$

In case (b) let x_1 be the deflection at the centre. Then taking moments on one side of the centre, the bending moment at this point

$$= \frac{Wl}{4} + Px_1$$

—*i.e.*, M has increased by Px_1.

Case (c) has a tensile end load, and in the same way

$$M = \frac{Wl}{4} - Px_2$$

showing M has decreased by Px_2.

x_1 is greater and x_2 less than x, due to the increased and decreased bending moments respectively.

This effect of end load on the stress distribution across a symmetrical section may readily be seen by reference to Fig. 128A.

   (a) shows the stress distribution due to pure bending,

   (b) the stress distribution due to end load only.
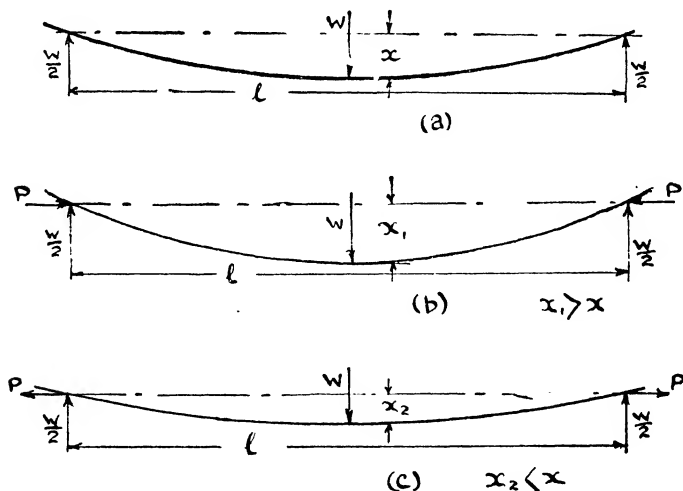
F<small>IG</small>. 128

Assuming the top part of the beam is in tension,

   (c) shows the increase in bending stress and the addition of direct stress due to compressive end load. The dotted line shows the original " pure bending " diagram for comparison.

   (d) shows the decrease in bending stress and the addition of direct stress due to tensile end-load.

It should be noticed that with compression the maximum stress is greatly increased and occurs on the compressive side, whilst with tension

TENSILE STRESS

COMPRESSIVE STRESS

$\frac{wl}{4Z}$

(a)

$\frac{P}{A}$

(b)

$\frac{wl}{4Z} + \frac{P_N}{Z} - \frac{P}{A}$

$\frac{P_N}{Z}$

$\frac{P}{A}$

$\frac{wl}{4Z} + \frac{P_N}{Z} + \frac{P}{A}$

(c)

$\frac{wl}{4Z} - \frac{P_N}{Z} + \frac{P}{A}$

$\frac{P_N}{Z}$

$\frac{P}{A}$

$\frac{wl}{4Z} - \frac{P_N}{Z} - \frac{P}{A}$

(d).
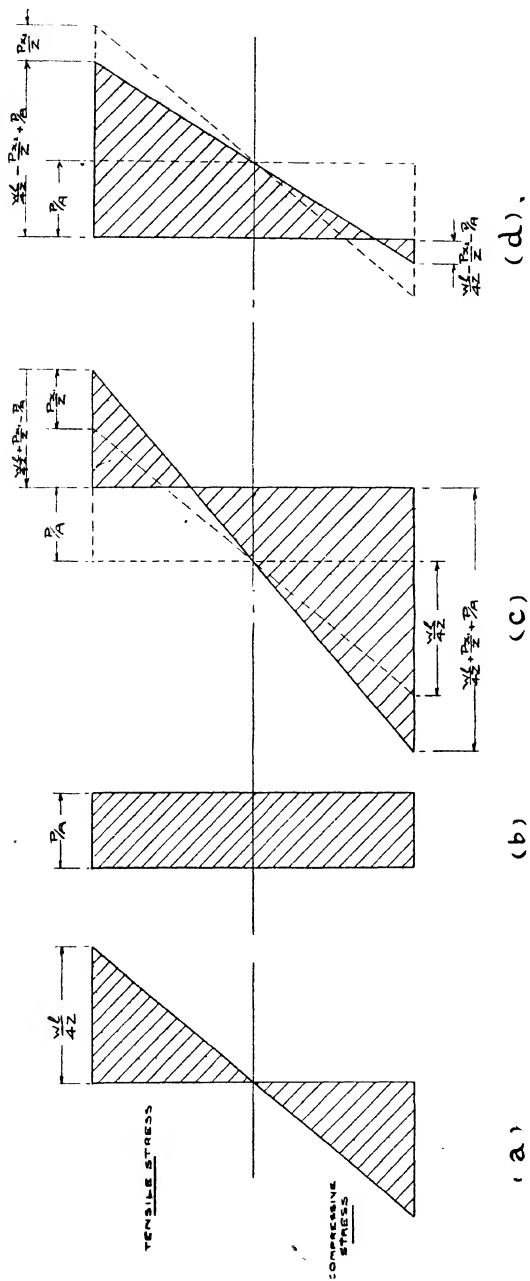
Fig. 128A

the maximum stress is only comparatively slightly increased, and occurs on the tensile side. In both cases it will be seen there is stress at the neutral axis.

The above results may be very easily illustrated by taking a long ruler loaded at the centre, and, using the hands as supports, first compress it and then put a pull in it. It will be seen that the deflection or strain, and therefore the bending stress, is increased in the first instance and decreased in the second.

It will be seen from the foregoing that a beam with a compressive end load will have to be made much larger and heavier than a similar one having a tensile or no end load.

This is a very important consideration in determining the arrangement of bracing.

The calculation of the bending moment of beams with end loads is a too difficult mathematical operation for inclusion in this book.

Continuous Beams.

A beam which rests on more than two supports is called a Continuous Beam.

The difficulty in solving a continuous beam is to obtain the reactions at the supports.

We will take a simple case first, and consider the general case later.

Let ABC (Fig. 129) be a beam of length l resting on three supports, one at each end and the other at the centre, and let it carry a load W uniformly distributed over the length with an intensity of w per unit length.

First suppose the support B has been removed ; the deflection at B then equals

$$\frac{5Wl^3}{384\,EI}$$

—*i.e.*, the deflection for a beam supported at the ends and having a uniformly distributed load W.

Now, due to support B, there is no deflection at B, therefore the reaction Q must be sufficient to deflect the beam upwards an amount equal to the deflection downwards when the support B was absent.

The deflection at B due to Q equals

$$\frac{Ql^3}{48\,EI}$$

—*i.e.*, the deflection for a beam supported at the ends and having a concentrated load Q at the centre.

Deflection downwards due to W = Deflection upwards due to Q.

$$\frac{5\,Wl^3}{384\,EI} = \frac{Ql^3}{48\,EI}$$

$$\tfrac{5}{8}\,W = Q.$$

Now $W = P+Q+R$

$P = R$

$\therefore W = 2P+Q$

$W = 2P+\tfrac{5}{8}W$

$P = \tfrac{3}{16}\,W = R.$
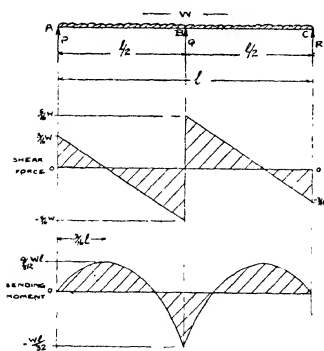
Fig. 129

Having obtained the reactions, the shear force and bending moment diagrams may be drawn in the usual manner.

The General Case.

Let AB and BC (Fig. 130) be two adjacent spans of a continuous beam carrying any kind of vertical loading. It is assumed that the beam is of uniform material and cross-section—*i.e.*, E and I are constant ; also that the supports are, and remain, on the same level.
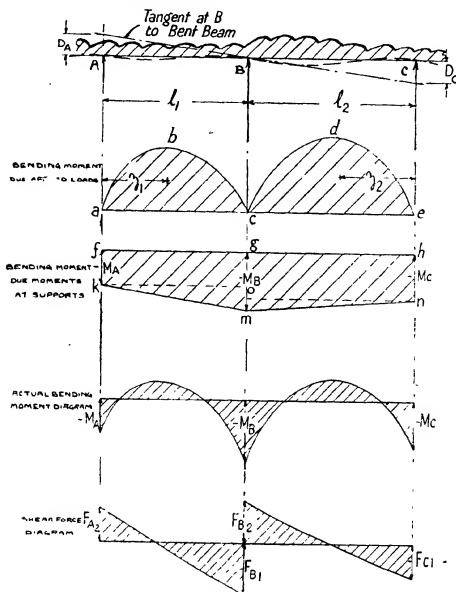
FIG. 130

Due to the continuity there will be unknown bending moments at each support A, B and C ; let these equal M_A, M_B and M_C respectively.

Consider the span AB.

The bending moment at any point on AB will equal the algebraic sum of the bending moment due to the applied loads on AB, considered as a beam freely supported at the ends, and the bending moment due to M_A and M_B. The bending moment diagram due to M_A and M_B may be found by considering a simple beam

FIG. 131

loaded at each end and overhanging its supports such that the bending moments at the supports are M_A and M_B respectively. Thus in Fig. 131.
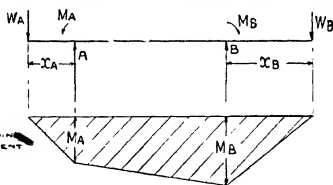
$$M_A = -W_A x_A$$
$$M_B = -W_B x_B$$

It will be seen that between A and B the bending moment diagram varies as a straight line from M_A to M_B.

Let abc be the bending moment diagram due to the applied loads, and $fgmk$ the bending moment diagram due to the bending moments at the supports. The actual bending moment diagram will equal the algebraic sum of these.

Let z_1 equal the distance of the centroid of the bending moment diagram *abc* from A, and let its area equal a_1.

Considering the tangent at B, as the original line from which to measure deflections—

Deflections at A

$$= D_A = \frac{1}{EI} \times \text{Moment of bending moment diagram between } A \text{ and } B \text{ about } A.$$

$$= \frac{1}{EI} \times \text{Moment of B.M.D. } fgmk + \text{Moment of B.M.D. } abc.$$

$$= \frac{1}{EI} \left[\text{Moment of B.M.D. } fgok + \text{Moment of B.M.D. } kom + \text{Moment of B.M.D. } abc \right]$$

$$= \frac{1}{EI} \left[M_A l_1 \times \frac{l_1}{2} + \frac{(M_B - M_A)}{2} l_1 \times \frac{2}{3} l_1 + a_1 z_1 \right]$$

$$= \frac{1}{EI} \left[\frac{M_A l_1^2}{2} + \frac{(M_B - M_A)}{3} l_1^2 + a_1 z_1 \right]$$

$$= \frac{1}{EI} \left(\frac{M_A l_1^2}{6} + \frac{M_B l_1^2}{3} + a_1 z_1 \right)$$

In the same way

Deflection at $C = D_C = \frac{1}{EI} \left(\frac{M_C l_2^2}{6} + \frac{M_B l_2^2}{3} + a_2 z_2 \right)$

where $a_2 =$ area of bending moment diagram *cde*

and $z_2 =$ distance of centroid of *cde* from C.

D_A and D_C must be of opposite sign, and from similar triangles—

$$\frac{D_A}{l_1} = \frac{-D_C}{l_2}$$

$$\frac{1}{l_1 EI} \left(\frac{M_A l_1^2}{6} + \frac{M_B l_1^2}{3} + a_1 z_1 \right) = -\frac{1}{l_2 EI} \left(\frac{M_C l_2^2}{6} + \frac{M_B l_2^2}{3} + a_2 z_2 \right)$$

$$\frac{M_A l_1}{6} + \frac{M_B l_1}{3} + \frac{a_1 z_1}{l_1} = -\frac{M_C l_2}{6} - \frac{M_B l_2}{3} - \frac{a_2 z_2}{l_2}$$

$$M_A l_1 + 2 M_B l_1 + \frac{6 a_1 z_1}{l_1} + M_C l_2 + 2 M_B l_2 + \frac{6 a_2 z_2}{l_2} = 0$$

$$M_A l_1 + 2 M_B (l_1 + l_2) + M_C l_2 + 6 \left(\frac{a_1 z_1}{l_1} + \frac{a_2 z_2}{l_2} \right) = 0.$$

This is known as the general equation of three moments.

In the case of a beam with a uniformly distributed load of w_1 per unit length on span AB, and of w_2 per unit length on span BC,

$$a_1 = \frac{2}{3} \frac{w_1 l_1^2}{8} \times l_1$$

$$= \frac{w_1 l_1^3}{12}$$

$$a_2 = \frac{w_2 l_2^3}{12}$$

$$z_1 = \frac{l_1}{2}$$

$$z_2 = \frac{l_2}{2}$$

and the equation of three moments becomes

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 + \left(\frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4} \right) = 0$$

Shearing Force and Reactions at the Supports.

It was found in Chapter X that the difference between the bending moment at any two sections of a beam is equal to the area of the shearing force diagram between these two sections.

The bending moment at B is thus equal to the bending moment at A plus the area of the shearing force diagram between A and B.

Let F_{B1} and F_{C1} be the shearing forces immediately to the left of B and C respectively, and F_{A2} and F_{B2} be the shearing forces immediately to the right of A and B respectively. Let W_1 and W_2 be the sum of the applied loads on the spans AB and BC respectively, and Z_1 and Z_2 be the horizontal distance of the C.G. of W_1 and W_2 from A and C respectively.

Then the area of the shearing force diagram between A and B

$$= F_{B1} \times l_1 + W_1 Z_1$$

and $M_B = M_A + F_{B1} l + W_1 Z_1$

$$-F_{B1} = \frac{M_A - M_B + W_1 Z_1}{l_1}$$

In the same way

$$F_{B2} = \frac{M_C - M_B + W_2 Z_2}{l_2}$$

It will be seen from the shearing force diagram that the reaction at B

$$R_B = -F_{B1} + F_{B2}$$

$$= \frac{M_A - M_B + W_1 Z_1}{l_1} + \frac{M_C - M_B + W_2 Z_2}{l_2}$$

We have now sufficient data to solve any continuous beam, with vertical loading. In using the formulæ the signs of bending moment and shearing force must be taken into account. Usually the bending moments at the supports and the shearing forces immediately to the left of the supports will be negative.

Method of Solving a Continuous Beam.

The method of solving a continuous beam with the formulæ just obtained is best illustrated by an example.

Consider the beam, Fig. 132. It is supported at A, B and C, and carries a uniformly distributed load of 50 lb./ft. on the span BC, and 40 lb./ft. on the span AB and the overhanging portion.

Using the same symbols as in the formulæ,

$$l_1 = 12 \text{ ft}, \quad l_2 = 10 \text{ ft},$$

$$w_1 = 40 \text{ lb./ft.}, \qquad w_2 = 50 \text{ lb./ft.}$$

$$M_A = \text{bending moment due to overhang}$$

$$= -40 \times 4 \times 2 = -320 \text{ ft. lb.}$$

$$M_C = 0$$
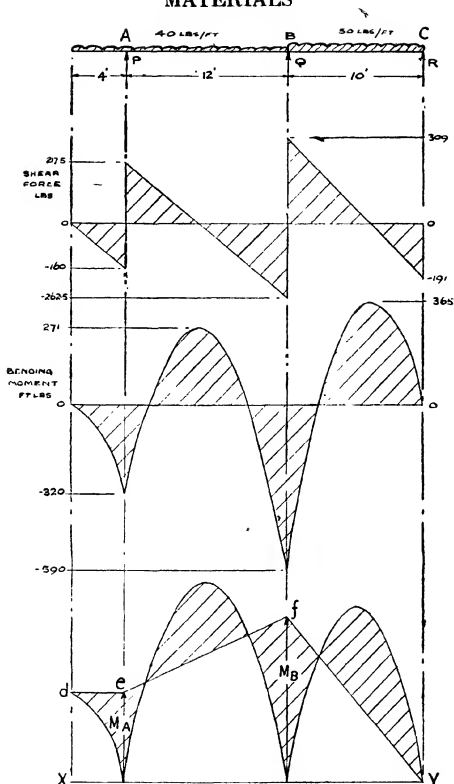
FIG. 132

The three moment equation for a uniformly distributed load is

$$M_A l_1 + 2 M_B(l_1 + l_2) + M_C l_2 + \left(\frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4}\right) = 0$$

$$-320 \times 12 + 2 M_B (12 + 10) + 0 + \left(\frac{40 \times 12^3}{4} + \frac{50 \times 10^3}{4}\right) = 0$$

$$-3840 + 44 M_B + 17280 + 12500 = 0$$

$$M_B = \frac{3840 - 17280 - 12500}{44}$$

$$= \frac{-25940}{44}$$

$$= -590 \text{ ft. lb.}$$

$$R_B = \frac{M_A - M_B + W_1 Z_1}{l_1} + \frac{M_C - M_B + W_2 Z_2}{l_2}$$

$$W_1 = 40 \times 12 = 480 \text{ lb.} \qquad Z_1 = 6 \text{ ft.}$$
$$W_2 = 50 \times 10 = 500 \text{ lb.} \qquad Z_2 = 5 \text{ ft.}$$

$$R_B = Q = \frac{-320 + 590 + (480 \times 6)}{12} + \frac{590 + (500 \times 5)}{10}$$

$$= 262 \cdot 5 + 309$$

$$= 571 \cdot 5 \text{ lb.}$$

The other two reactions, P and R, may be found by applying the conditions of equilibrium.

Taking moments about C,

$$10 \times 571 \cdot 5 + 22P - 50 \times 10 \times 5 - 40 \times 16 \times 18 = 0$$

$$P = \frac{2500 + 11520 - 5715}{22}$$

$$= 377 \cdot 5 \text{ lb.}$$

$$R + 571 \cdot 5 + 377 \cdot 5 - 50 \times 10 - 40 \times 16 = 0$$

$$R = 500 + 640 - 571 \cdot 5 - 377 \cdot 5$$

$$= 191 \text{ lb.}$$

The shearing force and bending moment diagrams may now be drawn in the usual manner, or the bending moment diagram may be drawn by drawing the bending moment diagram due to the moments at the supports, and the bending moment diagram due to the applied loads on each span considered as separate freely supported beams, and summing the two diagrams graphically. The diagram drawn by the latter method is shown in the second bending moment diagram in Fig. 132. The two diagrams are drawn on the base XY, and the resulting diagram is the shaded area. In this case the line of zero bending moment is $defY$. Although this diagram is not so easy to read, it has the advantage that it may be drawn without finding the reactions at the supports.

The Effect of Continuity.

We will study the effect of continuity in the following simple example : A continuous beam (Fig. 133) rests on five supports covering four equal spans of 10 ft. each, and is loaded with a concentrated load of 2 tons at the centre of each span.

$$M_A = M_E = 0$$
$$M_B = M_D \text{ from symmetry}$$
$$a_1 = a_2 = a_3 = a_4 = \frac{5 \times 10}{2} = 25$$
$$z_1 = z_2 = z_3 = z_4 = 5.$$

Apply equation of three moments to span ABC.

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 + 6 \left(\frac{a_1 z_1}{l_1} + \frac{a_2 z_2}{l_2} \right) = 0$$

$$0 + 2M_B (10 + 10) + M_C 10 + 6 \left(\frac{25 \times 5}{10} + \frac{25 \times 5}{10} \right) = 0$$

$$40M_B + 10M_C + 150 = 0$$

$$4 M_B + M_C + 15 = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Now apply the equation of three moments to spans BCD.

$$M_B l_2 + 2M_C (l_2 + l_3) + M_D l_3 + 6 \left(\frac{a_2 z_2}{l_2} + \frac{a_3 z_3}{l_3} \right) = 0$$

$$M_B 10 + 2M_C (10 + 10) + M_B 10 + 150 = 0 \quad (\text{Since } M_B = M_D)$$

$$20 M_B + 40 M_C + 150 = 0$$

$$4 M_B + 8 M_C + 30 = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Subtract equation (1) from (2).

$$7\,M_c + 15 = 0$$

$$M_c = \frac{-15}{7}$$

$$= -2\frac{1}{7} \text{ ft. tons.}$$
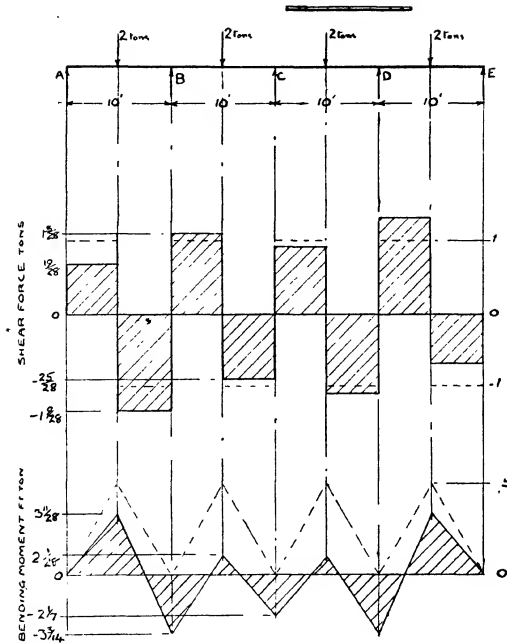
FIG. 133

Substitute value of M_c in (1).

$$4\,M_B \div 2\frac{1}{7} + 15 = 0$$

$$M_B = \frac{-90}{4 \times 7}$$

$$= -3\frac{3}{14} \text{ ft. tons.}$$

$R_B = R_D$ and $R_A = R_E$ from symmetry.

$$R_B = \frac{M_A - M_B + W_1 Z_1}{l_1} + \frac{M_C - M_B + W_2 Z_2}{l_2}$$

$$= \frac{0 + 3\frac{3}{14} + (2 \times 5)}{10} + \frac{-2\frac{1}{7} + 3\frac{3}{14} + (2 \times 5)}{10}$$

$$= 2\frac{3}{7} \text{ tons.}$$

$$R_c = \frac{M_B - M_0 + W_2 Z_2}{l_1} + \frac{M_D - M_c + W_3 Z_3}{l_3}$$

$$= \frac{-3\frac{3}{14} + 2\frac{1}{7} + (2 \times 5)}{10} + \frac{-3\frac{3}{14} + 2\frac{1}{7} + (2 \times 5)}{10}$$

$$= 1\frac{11}{14} \text{ tons.}$$

From conditions of equilibrium

$$R_A + R_B + R_C + R_D + R_E - 8 = 0$$

$$2R_A + 2R_B + R_C - 8 = 0$$

$$2R_A + 2 \times 2\frac{3}{7} + 1\frac{11}{14} - 8 = 0$$

$$R_A = \frac{8 - 4\frac{6}{7} - 1\frac{11}{14}}{2}$$

$$= \frac{19}{28} \text{ ton.}$$

The shearing force and bending moment diagrams are shown shaded in the figure, and superimposed by the dotted line are the diagrams for a similarly loaded beam pin-jointed at the supports—*i.e.*, not continuous.

It will be seen that the maximum shearing force is increased in the case of the continuous beam, and this will mean slightly heavier webs in such beams as thin metal spars.

Far more important is the effect of continuity on the bending moment. This is considerably reduced in the middle of the spans, and increased, but of opposite sign, near the supports. The maximum bending moment which occurs at the supports is less than for the pin-jointed beam. Thus the continuous beam as a whole may be made lighter.

Due to the bending moment being smaller and changing sign, the moment of the bending moment diagram will be considerably reduced; thus the deflection, which is proportional to the moment of the bending moment diagram, will be greatly reduced. This is a very great advantage in the case of aeroplane spars with compressive end loads, for the effect of the end load in increasing the bending moment is reduced with reduction of deflection.

The disadvantage of continuity is that if the supports are not in line, due to subsidence, faulty construction or, in an aeroplane, bad rigging, the bending moments may be considerably altered, and thus may increase the stresses over the maximum the beam is designed to take.

Note.—The same conclusions hold for continuous beams carrying distributed loads.

EXAMPLE 1.—A steel cantilever 10 ft. long carries a load of 2·6 tons uniformly distributed over the whole length. If the beam is of rectangular section 4 inches deep and 3 inches wide, find the maximum stress in the beam and the deflection and slope at the free end.

$$E \text{ for steel } = 13{,}000 \text{ tons/sq. in.}$$

Neglect the weight of the beam.

$$I = \frac{bd^3}{12} = \frac{3 \times 4^3}{12} = 16 \text{ inches}^4.$$

$$Z = \frac{I}{y} = \frac{16}{2} = 8 \text{ inches}^3.$$

$$M = 2 \cdot 6 \times 5 = 13 \text{ ft. tons.}$$

$$= 13 \times 12 \text{ in. tons.}$$

$$\text{Max. stress } f = \frac{M}{Z} = \frac{13 \times 12}{8}$$

$$= 19 \cdot 5 \text{ tons/sq. inch.}$$

$$\text{Deflection} = \frac{Wl^3}{8EI}$$

$$= \frac{2 \cdot 6(10 \times 12)^3}{8 \times 13000 \times 16}$$

$$= 2 \cdot 7 \text{ inches.}$$

$$\text{Slope} = \frac{1}{EI} \cdot \frac{Wl}{2} \times \frac{l}{3}$$

$$= \frac{2 \cdot 6 \times 120^2}{13000 \times 16 \times 6}$$

$$= \cdot 03 \text{ radians.}$$

EXAMPLE 2.—What is the deflection of a steel tubular beam 4 inches outside diameter and $3\frac{1}{2}$ inches inside diameter when it carries a uniform load of 800 lb./ft., if it is 12 feet long and is freely supported at the ends ? Neglect the weight of the beam.

$$I = \frac{\pi}{64} (d^4 - d_1{}^4)$$

$$= \frac{\pi}{64} (4^4 - 3 \cdot 5^4)$$

$$= \frac{\pi}{64} \times 106$$

$$= 5 \cdot 2 \text{ inches}^4.$$

$$w = 800 \text{ lb./ft.}$$

$$= \frac{800}{12} \text{ lb./in.}$$

$$\text{Deflection} = \frac{5wl^4}{384EI}$$

$$= \frac{5 \times 800 \times (12 \times 12)^4}{384 \times 12 \times 30000000 \times 5 \cdot 2}$$

$$= \frac{5 \times 800 \times 420000000}{384 \times 12 \times 30000000 \times 5 \cdot 2}$$

$$= 2 \cdot 3 \text{ inches.}$$

EXAMPLE 3.—A steel cantilever 15 feet long carries a load of 1,000 lb. at the free end. The section is rectangular, 5 inches deep and 3 inches wide. What is the deflection at the free end? Weight of steel = 0·283 lb./cu. inch.

$$\text{Weight of beam} = 5 \times 3 \times 15 \times 12 \times 0{\cdot}283$$
$$= 764 \text{ lb.}$$

$$I = \frac{bd^3}{12} = \frac{3 {\cdot} 5^3}{12}$$
$$= 31{\cdot}25 \text{ inches}^4.$$

$$\text{Deflection} = \frac{l^3}{EI}\left(\frac{P}{3} + \frac{W}{8}\right)$$
$$= \frac{(15 \times 12)^3}{30000000 \times 31{\cdot}25}\left(\frac{1000}{3} + \frac{764}{8}\right)$$
$$= \frac{5832000 \times 429}{30000000 \times 31{\cdot}25}$$
$$= 2{\cdot}67 \text{ inches.}$$

EXAMPLE 4.—A timber beam 8 feet long is fixed at one end, and supported freely at the other, such that the two ends are on the same level. It normally carries a load of 100 lb. evenly distributed over the whole length, and a load factor of 4 is to be used.

If the beam is to be of I section 2 inches deep and with ½ inch thick flanges and web, find the necessary breadth. Take an allowable stress of 4,000 lb./sq. inch.
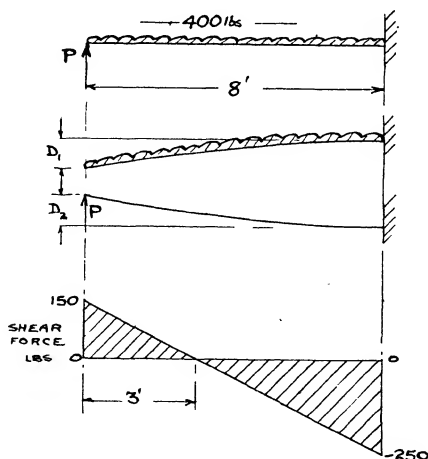
FIG. 134

Referring to Fig. 134, the deflection down due to the applied load is overcome by the load in the support such that there is no deflection at the supports.

i.e., Deflection due to *W* alone —Deflection due to *P* alone =0.

$$D_1 - D_2 = 0$$

$$\frac{Wl^3}{8EI} - \frac{Pl^3}{3EI} = 0$$

$$\frac{W}{8} - \frac{P}{3} = 0$$

$$P = \tfrac{3}{8} W.$$

$$W = 100 \times 4 = 400 \text{ lb.}$$

$$w = \frac{400}{8} = 50 \text{ lb./ft.}$$

$$P = \tfrac{3}{8} \times 400 = 150 \text{ lb.}$$

Draw shearing-force diagram.

It is seen from the shearing force diagram that the maximum bending moment occurs either at the fixed end or 3 feet from the free end—*i.e.* where shearing force is zero. B.M. at fixed end

$$= 8 \times 150 - 8 \times 50 \times 4$$
$$= 1200 - 1600$$
$$= -400 \text{ ft. lb.}$$

B.M. at 3 feet from free end

$$= 3 \times 150 - 3 \times 50 \times 1\cdot5$$
$$= 450 - 225$$
$$= 225 \text{ ft. lb.}$$

Maximum bending moment occurs at the fixed end = 400 ft. lb.

$$Z = \frac{M}{f} = \frac{400 \times 12}{4000}$$

$$= 1\cdot2 \text{ in.}^3$$

$$Z = \frac{bd^3 - b_1 d_1^3}{6d}$$

$$= \frac{bd^3 - (b-t)d_1^3}{6d}$$

$$= \frac{b(d^3 - d_1^3) + t d_1^3}{6d}$$

$$b = \frac{6dZ - t d_1^3}{(d^3 - d_1^3)}$$

$$= \frac{6 \times 2 \times 1\cdot2 - 0\cdot5}{8 - 1}$$

$$= \frac{13\cdot9}{7}$$

$$= 2 \text{ inches} = \text{breadth.}$$

CHAPTER XIII

STRUTS

Struts are members loaded in compression along their longitudinal axis. The load tends to increase any curvature, causing a strut to fail by bending as well as direct compression, unless it is very short, at a load very much less than the same member would withstand in tension.

We may divide struts into three types—viz. :—

(1) Short struts ; those whose length is not greater than about twice their least width. These are subjected to pure compression, so that

$$\text{Stress} = \frac{\text{Load}}{\text{Area of cross-section}}$$

(2) Medium-length struts ; those which are subjected to combined bending and compression, but do not start to buckle until the elastic limit of the material has been reached.

(3) Slender struts ; those which are very long compared with their least radius of gyration. They may be considered to fail entirely by bending, and buckle before the elastic limit of the material has been reached. Their strength does not depend upon the strength of the material, but upon the modulus of elasticity, cross-section, and length.

Buckling of Slender Struts.

Consider the slender strut ACB (Fig. 135A) pin-jointed at its ends, so that it is free to bend throughout its length, and let it be perfectly straight when unloaded. Apply an axial load P_1, less than the buckling load, and the strut will remain straight. Now bend the strut by means of a lateral force W at the centre C, such that the deflection at C equals d_1. If the applied loads are not sufficient to make the strut collapse, it is held in equilibrium so that the bending moment equals the moment of resistance.

The bending moment at $C = P_1 d_1 + \frac{Wl}{4}$

—*i.e.*, the moment of P_1 and $\frac{W}{2}$ about C.

Now remove W, and the bending moment is reduced to $P_1 a_1$, which is less than the moment of resistance, and the strut is straightened. If, however, we increase P_1 as we remove W, so that the bending moment is constant, the strut will remain bowed. When W is removed the whole of the bending moment will be due to P_1. Any decrease of P_1 will reduce the bending moment and the strut will straighten, whilst any increase of P_1 will increase the bending moment above the moment of resistance and the strut will, in consequence, bend further. The increased deflection of the strut will cause an increase in the bending moment due to P_1, and the increased curvature will cause increased strain at C, resulting in an increase of the tensile and compressive reactions in the material, and of the moment of resistance. However, the moment of resistance will not increase as rapidly as the bending moment, with the result that the bending moment will always be greater, causing the strut to continue bowing until failure occurs.

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Such a failure is said to be caused by elastic instability.   The value of P_1 that just kept the strut in equilibrium is, therefore, the maximum load the strut can .withstand, and is called the buckling load.

We have seen that buckling occurs when the bending moment exceeds the moment of resistance ; let us now consider on what this depends.   For a given curvature of the strut the deflection depends upon the length : the greater the length the greater the deflection and so the greater the bending moment for a given load or the smaller the load for a given deflection.

Again for a given curvature, the strain at any point depends upon its distance from the axis.   For example, the strain of the material
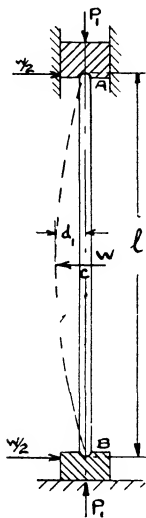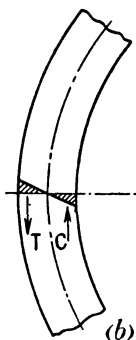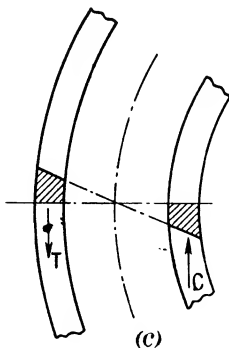
FIG. 135A   FIG. 135B   FIG. 135C

of the solid strut, Fig. 135B, is less than the average strain in the material in the hollow strut, Fig. 135C, as shown by the shading.   It follows that the stress and the reactions T and C are greater where the material is at a distance from the axis.   Not only are the reactions greater, but, as can be seen, their moment is greater, giving a bigger buckling load.   Again, materials of high modulus of elasticity produce larger reactions, for a given strain, than those of low modulus.

The above should make it clear that the buckling load depends upon—

(1) the length,
(2) the cross-section,
(3) the modulus of elasticity of the material.

Now let us see why the strength of a slender strut does not depend upon the strength of its material, whilst that of a medium-length strut does.   Fig. 135D shows bowed slender and medium-length struts superimposed.   The stress distribution at the centre is shown shaded. If we assume that the maximum stress in the medium-length strut is 50 ton/sq. in.,.that in the slender strut will be only 10 ton/sq. in. because its width is only one-fifth.   Let us also assume that the maximum stress the material will withstand is 40 ton/sq. in.

The width and stress in the slender strut being so small, it will have a small moment of resistance, and we may assume it elastically unstable under the load. Note that, although it is failing, the stress is below 40 ton/sq. in., but it will go on bowing and increasing the curvature

Slender Strut

Medium-length Strut

Stress = 50 ton/sq. in.
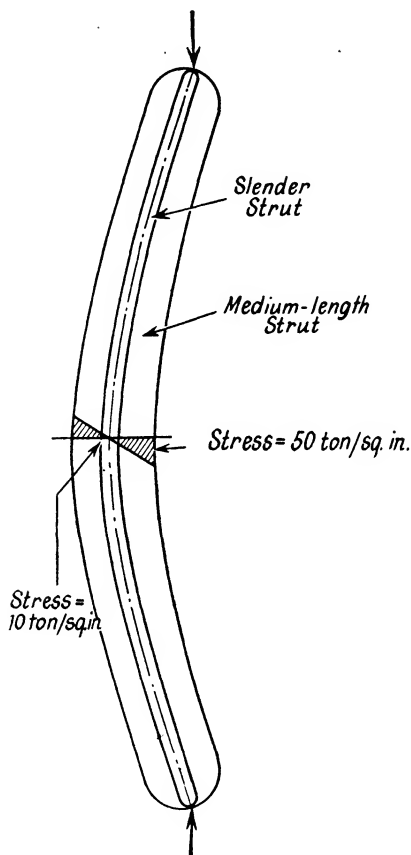
Stress = 10 ton/sq.in.

FIG. 135D

until a stress of 40 ton/sq. in. is reached, when the material will fail. In effect the strut has failed before the material.

The medium-length strut having a much greater stress and width will have a large moment of resistance, and will not become elastically unstable until the material has failed at 40 ton/sq. in.

It follows that while it is an advantage to use strong material for a medium-length strut, which does not fail before the elastic limit of the material has been reached, a slender strut, that becomes elastically unstable below the elastic limit, gains no advantage when made of material having a greater proof stress than the stress in the material when elastic instability occurs.

Thin Metal Struts.

We may have a compression member which is not slender, when considered as a whole, but parts of it may fail by elastic instability. Consider the thin steel strut, Fig. 135E. It is not a slender strut, but the outside edge will bow under load, independently of the rest, and become elastically unstable at a very low stress.

If now the non-rigid portion is shortened, as in Fig. 135F, the rigid part will prevent a single bow, but it will form the waves shown dotted ; each wave being in effect a small slender elastically unstable strut. If the non-rigid portion be further shortened the waves will get shorter. Now imagine a portion the length of a wave cut out, as in Fig. 135G,
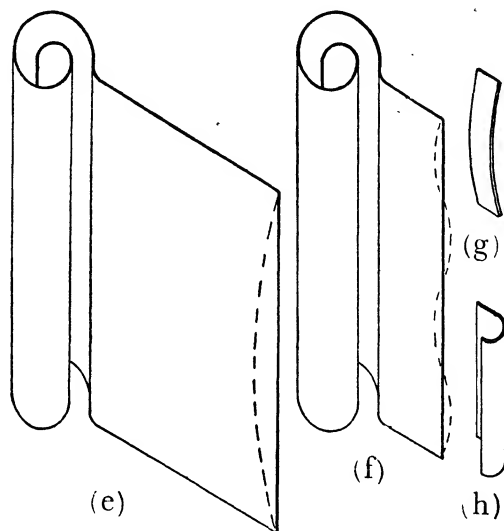
FIG. 135. E, F, G, H

and we see it is a slender strut. As the waves get shorter these little "independent" struts become less slender, because their length-thickness ratio is decreasing, until they become medium-length struts. If then the straight portion is sufficiently narrow, elastic instability will not occur. If instead of making the straight portion (Fig. 135F) narrower we put a small radius down the outside edge, elastic instability will not occur, as the previous waved parts now form medium-length struts, as illustrated in Fig. 135H.

Take two pieces of paper, 6 in. × 4 in. and 12 in. × 4 in., and make them into two cylindrical struts, by gumming together their outer edges. Load them carefully with books, and it will be found that the small one will carry as much as, or a greater load than, the large one. If a thick material had been used the large diameter strut would have carried twice the load of the small one, but with a thin material, elastic instability occurs at a low stress, depending on the diameter-thickness ratio.

It has been shown that we require the material at a distance from the axis for high strength-weight ratio. It would seem to follow that

as we increase the diameter of a tubular strut and decrease the wall thickness, to give constant cross-sectional area, the strength would increase. This is true up to a diameter-thickness ratio of about 100. Above this limit the strut gets weaker because the walls are so thin compared to their curvature that they can fail, independently of the strut as a whole, by elastic instability.

If the walls are corrugated the tendency to elastic instability is reduced, as may be seen from the results of an experiment on steel struts, illustrated in Fig. 136. Note that the proof stress of the material is 65 ton /sq.in., but that the uncorrugated strut only develops a stress
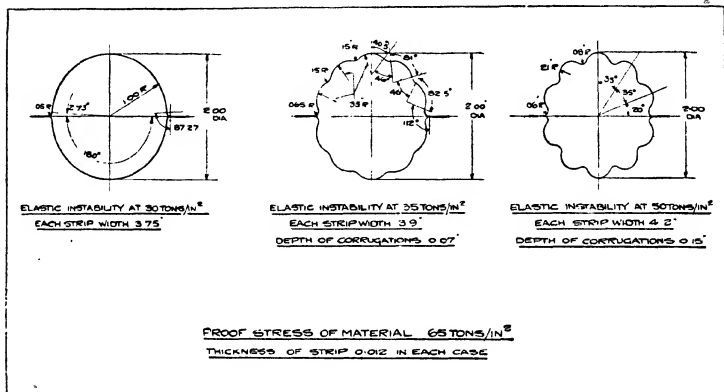
FIG. 136

of 30 ton/sq. in. before elastic instability sets in, and the strut fails. As the corrugations are made deeper so the stress developed increases. There is no advantage in using steel of 65 ton/sq. in. proof stress for any of these struts. Strut (a) would be the same strength if a cheaper 30 ton/sq. in. steel were used, strut (b) a 35 ton/sq. in. steel, and strut (c) a 50 ton/sq. in. steel. Note the better the steel the greater the corrugations required to take full advantage of its strength.

Euler's Formula.

There are several strut formulæ, some theoretical and some empirical, for obtaining the buckling loads of struts. Euler's formula, which is used for slender struts, states that

$$ P = \frac{\pi^2 E I}{l^2} $$

where P = the buckling load,

E = Young's modulus of the material,

I = least moment of inertia of the section about any axis through the centroid,

l = length of pin-jointed strut.

The mathematics involved in the proof of this, and in fact all the strut formulæ, is too advanced for this book. We may, however, obtain an approximate proof which will show the principles involved and the limitation of its use.

Consider a strut ACB (Fig. 137) which is held in equilibrium by the buckling load P, such that it is deflected a distance d at the centre C. The bending moment at $C = Pd$.

The bending moment at any other point will equal the product of P and the deflection at that point. This gives the bending moment diagram acb, which is nearly a parabola, and will be assumed to be one for this approximate proof. Actually it is a sine curve.
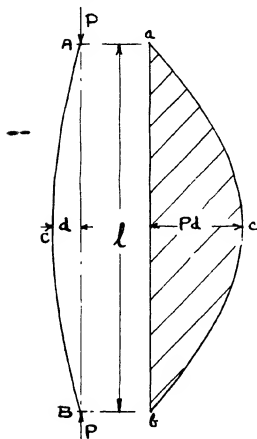
Deflection

$$d = \frac{1}{EI} \times \text{moment of } \tfrac{1}{2} \text{ bending moment diagram about } A.$$

$$d = \frac{1}{EI} \times \frac{2}{3} \frac{l}{2} Pd \times \frac{5}{8} \frac{l}{2}$$

$$d = \frac{5\ Pdl^2}{48\ EI}$$

$$P = \frac{48\ EI}{5l^2}$$

$$= \frac{9 \cdot 6\ EI}{l^2}$$

This result is very nearly Euler's, which gives

$$P = \frac{9 \cdot 87\ EI}{l^2}$$

Fig. 137

Euler's Formula may be written in terms of buckling load per unit area p, thus—

$$p = \frac{P}{A} = \frac{\pi^2 EI}{Al^2}$$

$$I = AK^2$$

where K = least Radius of Gyration.

$$p = \frac{\pi^2 EAK^2}{Al^2}$$

$$= \frac{\pi^2 EK^2}{l^2}$$

Limitations of Euler's Formula.

One defect in this formula is that it does not take into account the direct compressive stress on the strut. In a long strut this direct stress is small compared with the bending stress, but in a short strut the direct stress is large ; the formula may therefore only be used in the case of struts which are very long compared to their least transverse dimension. This is usually taken to mean if their slenderness ratio $\frac{l}{K}$ is over 130.

For example, consider a 2-inch diameter solid strut 20 inches long, made of mild steel having a compressive yield stress of 20 tons/sq. inch.

$$E = 13{,}000 \text{ tons/sq. inch.}$$

$$K = \frac{d}{4} \quad \text{(see Appendix I).}$$

$$p = \frac{\pi^2 E K^2}{l^2}$$

$$= \frac{9\cdot87 \times 13000 \times 4}{400 \times 16}$$

$$= 80 \text{ tons/sq. inch.}$$

As this is considerably greater than the direct yield stress of 20 tons/sq. inch, the strut will obviously fail long before the Euler value is reached.

Eccentricity of Loading.

An error arises in Euler's formula, due to the ideal conditions assumed not being fulfilled in practice.

No strut, however well made, is perfectly straight, or, if a tube, of equal thickness throughout. Nor can the assumption that the load line is along the axis of the strut when straight be realized in practice.

This initial curvature and eccentricity of loading causes the strut to have a bending moment and deflect under the smallest loads. As the load is increased the deflection increases until failure takes place at a load less than the Euler value.

Rankin's Formula.

A formula much used in general engineering (not aeronautical), which has the advantage of taking into account the direct compressive stress, is known as Rankin's formula—viz. :

$$P = \frac{f_c A}{1 + \dfrac{cl^2}{K^2}}$$

where f_c = ultimate compressive stress,

A = cross-sectional area,

c = a constant for a given material,

K = least radius of gyration.

Consider a straight column loaded eccentrically with a load P at a distance x from its axis (Fig. 138).

The maximum compressive stress f_c will equal the direct compressive stress f_1 plus the stress due to bending f_2.

Bending moment $= Px$

$$f_1 = \frac{P}{A}$$

$$f_2 = \frac{My}{I}$$

$$= \frac{Pxy}{I}$$

$$= \frac{Pxy}{AK^2}$$

$$f_c = f_1 + f_2$$

$$= \frac{P}{A} + \frac{Pxy}{AK^2}$$

$$= \frac{P}{A}\left(1 + \frac{xy}{K^2}\right)$$

$$P = \frac{f_c A}{1 + \dfrac{xy}{K^2}}$$

Now consider the strut ACB (Fig. 137) deflected a distance d at the centre.

In the same way as for the column just considered—

$$P = \frac{f_c A}{1 + \dfrac{dy}{K^2}}$$

From Euler : $$P = \frac{\pi^2 EI}{l^2}$$

$$Pd = \frac{\pi^2 EId}{l^2} \qquad Pd = M = \frac{f_c l}{y}$$

$$\frac{f_c I}{y} = \frac{\pi^2 EId}{l^2}$$

$$d = \frac{f_c l^2}{\pi^2 Ey}$$

$$d = c\,\frac{l^2}{y}$$

where $c =$ a constant for a given material,

$$= \frac{f_c}{\pi^2 E}, \text{ but usually determined by experiment.}$$

From above, $$P = \frac{f_c A}{1 + \dfrac{dy}{K^2}}$$

Substituting for d

$$P = \frac{f_c A}{1 + \dfrac{cl^2}{K^2}}$$

The value of c is usually taken as equal to

$\dfrac{1}{7500}$ for mild steel $f_c = 21$ tons/sq. inch.

$\dfrac{1}{9000}$ for wrought iron $f_0 = 16$ tons/sq. inch.

$\dfrac{1}{1600}$ for cast iron $f_c = 36$ tons/sq. inch.

$\dfrac{1}{2000}$ for timber $f_c = 1.8$ tons/sq. inch.

Rankin's formula may be used for short struts, as, unlike Euler's, it takes into account the direct compressive stress.   It is not, however,

accurate as it fails to take into account the initial curvature and eccentricity of loading.
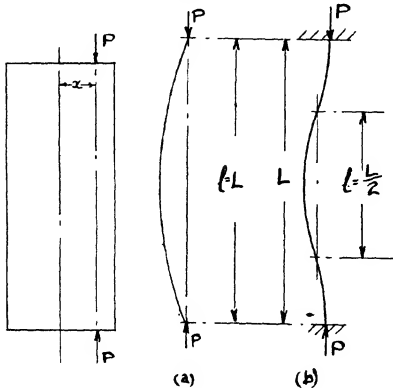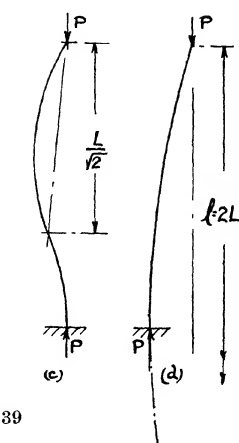
FIG. 138   FIG. 139

End Fixings.

So far we have only considered struts which are pin-jointed at their ends.

If the ends are jointed in any other way, l in the formula will not be the length of the strut, but the length of the equivalent pin-jointed strut.

The equivalent lengths of four standard cases are given below. L is the actual length of strut in each case.

Case Fig.	Type of Strut.				Equivalent Length (l).
a	Pin-jointed at each end	...	...	...	...L
b	Fixed at each end	...	...	...	... $L/2$
c	Fixed one end, pin-jointed other end		...	...	$\dfrac{L}{\sqrt{2}}$
d	Fixed one end, other end free	...	...	...	$2L$

We will not attempt to prove these figures, but they may be understood by reference to the buckling shapes shown in Fig. 139.
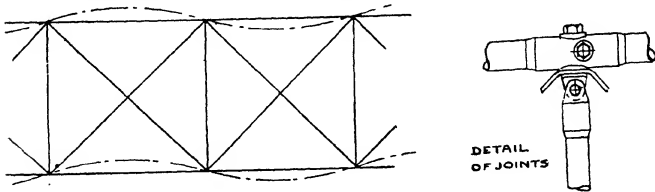
FIG. 140

Note.—A strut end can only be considered as fixed if the end-fixing is itself rigid and unable to move with the deflection of the strut— *e.g.*, the longerons of an aeroplane are not actually pin-jointed, but as they may deflect as in Fig. 140 they are considered to be pin-jointed.

Robertson's Formula.

Much work has been done to devise a formula which will take into account the initial curvature and eccentricity of loading.

In general engineering the generous factors of safety used are sufficient to cover any error on this account, but in aeronautical engineering a more accurate solution is desired.

A formula which gives extremely good results has been devised by Dr. Robertson, and is generally used for the design of wooden struts in aircraft.

This formula is—

$$p = \frac{P}{A} = \frac{p_y + (n+1)p_e}{2} - \sqrt{\left(\frac{p_y + (n+1)p_e}{2}\right)^2 - p_y p_e}$$

where p = buckling load per unit area,

P = buckling load,

A = cross-sectional area,

p_y = compressive yield-stress for steel or ultimate compressive stress for timber,

p_e = Euler buckling load per unit area = $\dfrac{\pi^2 E K^2}{l^2}$

n = 0·003 l/K,

l = length of pin-jointed strut,

K = least radius of gyration.

EXAMPLE.—Find the buckling load of a 2-inch diameter, 6-feet long pin-jointed steel strut.

The material has a compressive yield stress of 18 tons/sq. inch. and ultimate compressive stress of 21 tons/sq. inch.

(a) By Euler's formula.

(b) By Rankin's formula.

(c) By Robertson's formula.

(a) E = 13000 tons/sq. inch.

$A = \dfrac{\pi}{4} d^2$ = 3·14 sq. inch.

$K = \dfrac{d}{4}$ = ½ inch.

$p = \dfrac{\pi^2 E K^2}{l^2}$

$= \dfrac{9·87 \times 13000}{72 \times 72 \times 4}$

$= 6·2$ tons/sq. inch.

$P = pA = 6·2 \times 3·14$

$= 19·5$ tons.

(b)

$$P = \frac{f_c A}{1 + c \dfrac{l^2}{K^2}}$$

$$= \frac{21 \times 3 \cdot 14}{1 + \dfrac{72 \times 72 \times 4}{7500}}$$

$$= \frac{21 \times 3 \cdot 14}{3 \cdot 76}$$

$$= 17 \cdot 5 \text{ tons.}$$

(c)

From (a) $p_e = 6 \cdot 2$ tons/sq. inch.

$$n = 0 \cdot 003 \; l/K = 0 \cdot 003 \times 72 \times 2$$

$$= 0 \cdot 432.$$

$$p = \frac{p_y + (n+1)\,p_e}{2} - \sqrt{\left(\frac{p_v + (n+1)\,p_e}{2}\right)^2 - p_y p_e}$$

$$= \frac{18 + 1 \cdot 432 \times 6 \cdot 2}{2} - \sqrt{\left(\frac{18 + 1 \cdot 432 \times 6 \cdot 2}{2}\right)^2 - 18 \times 6 \cdot 2}$$

$$= 13 \cdot 44 - \sqrt{180 \cdot 6 - 111 \cdot 6}$$

$$= 13 \cdot 44 - \sqrt{69}$$

$$= 5.13 \text{ tons/sq. inch.}$$

$$P = pA$$

$$= 5 \cdot 13 \times 3 \cdot 14$$

$$= 16 \cdot 11 \text{ tons.}$$

These results show how the eccentricity of loading reduces the load the strut will bear, and that results (a) and (b) are not safe unless a margin of safety is used. (a) gives a particularly high load due to not taking into account the direct stress.

The use of Robertson's formula direct for determining sizes of struts for various conditions is rather laborious. It is therefore usual first to plot a graph of p against l/K for the material used.

An example for Grade B Spruce is given in Fig. 141. From this graph other graphs of buckling load against length for different sized and shaped sections may be readily plotted. Examples for square section struts are given in Fig. 142. From these graphs the necessary sizes of struts may be readily found—e.g., a square Grade B Spruce strut 2 ft. 6 in. long, carrying a load of 7,000 lb., would have to be $1\frac{3}{4}$ in. $\times 1\frac{3}{4}$ in.
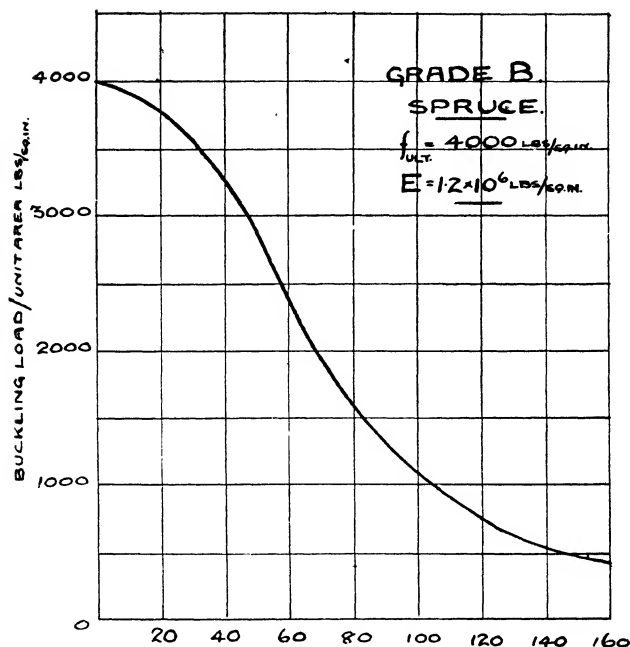
Fig. 141

Southwell's Formula.

A much more complicated formula, due to Prof. Southwell, is used for obtaining the strength of tubular metal struts in aircraft—viz.—

$$ P = pA = \frac{p_y A}{1 + \frac{d\delta}{2K^2} \operatorname{Sec}\left\{ \sqrt{\frac{p}{EK^2}} \times \frac{l}{2} \right\}} $$

where d = depth of section in plane of bending,

= outside diameter of circular tube,

δ = equivalent eccentricity of loading, for tubular struts this equals

$$ \frac{l}{600} + \frac{\text{internal diameter}}{40} $$

The other symbols are as for Robertson's formula.

This formula cannot be used directly, but graphs may be drawn, and the sizes of struts for various loads and lengths found from them.

The method of obtaining these graphs from this formula is far too involved for inclusion in this book. Those readers who wish to find the sizes of tubular struts for aircraft will find the graphs for steel and duralumin given in Appendix III, Vol. II.
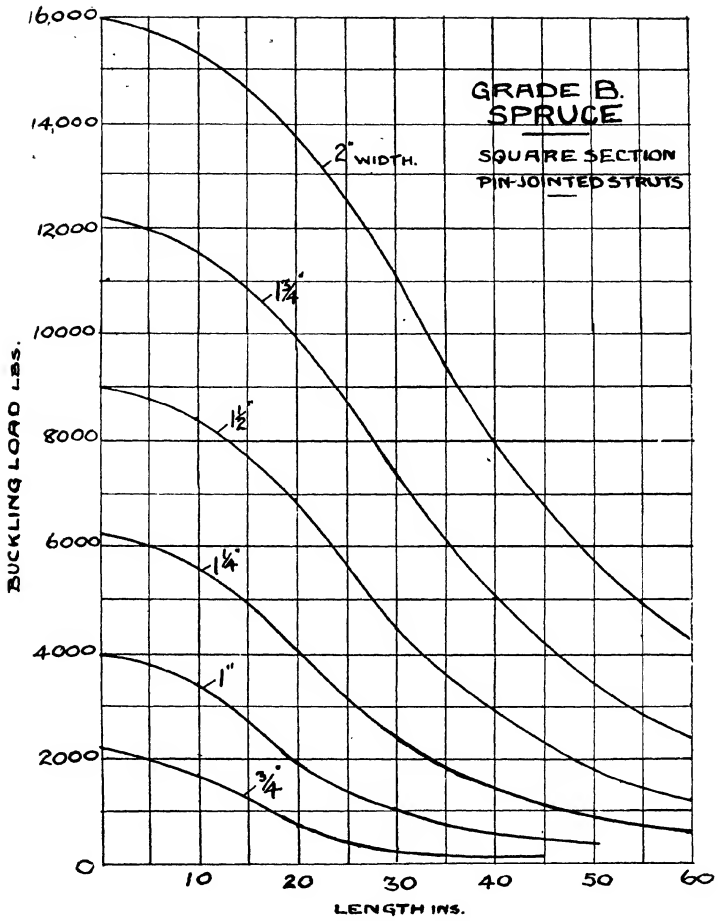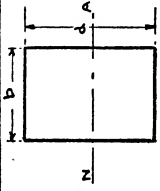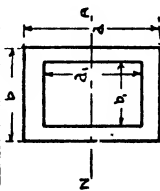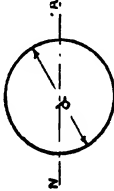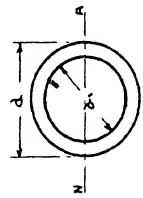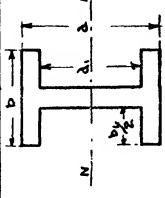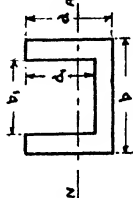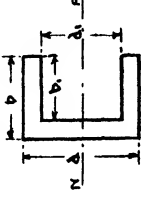
FIG. 142

APPENDIX I

Some Useful Constants of Standard Sections

SECTION	AREA A	Distance of Outermost Fibre from Neutral Axis y	Moment of Inertia about Neutral Axis I_{NA}	Modulus of Section $Z = \dfrac{I_{NA}}{y}$	Radius of Gyration $K = \sqrt{\dfrac{I_{NA}}{A}}$
	bd	$\dfrac{d}{2}$	$\dfrac{1}{12}\,bd^3$	$\dfrac{1}{6}\,bd^2$	$\dfrac{d}{\sqrt{12}}$
	$bd - b_1 d_1$	$\dfrac{d}{2}$	$\dfrac{bd^3 - b_1 d_1^3}{12}$	$\dfrac{bd^3 - b_1 d_1^3}{6d}$	$\sqrt{\dfrac{bd^3 - b_1 d_1^3}{12(bd - b_1 d_1)}}$
	$\dfrac{\pi}{4}\,d^2$	$\dfrac{d}{2}$	$\dfrac{\pi d^4}{64}$	$\dfrac{\pi d^3}{32}$	$\dfrac{d}{4}$

Section	Area	e	I	Section modulus	Radius of gyration
(hollow circle)	$\dfrac{\pi}{4}(d^2 - d_1^2)$	$\dfrac{d}{2}$	$\dfrac{\pi}{64}(d^4 - d_1^4)$	$\dfrac{\pi}{32}\dfrac{(d^4 - d_1^4)}{d}$	$\sqrt{\dfrac{d^2 + d_1^2}{4}}$
(I-section)	$bd - b_1 d_1$	$\dfrac{d}{2}$	$\dfrac{bd^3 - b_1 d_1^3}{12}$	$\dfrac{bd^3 - b_1 d_1^3}{6d}$	$\sqrt{\dfrac{bd^3 - b_1 d_1^3}{12(bd - b_1 d_1)}}$
(C-section)	$bd - b_1 d_1$	$\dfrac{bd^2 - b_1 d_1^2}{2(bd - b_1 d_1)}$	$\dfrac{bd^3 - b_1 d_1^3}{3} - \dfrac{(bd^2 - b_1 d_1^2)^2}{4(bd - b_1 d_1)}$	$\dfrac{2(bd - b_1 d_1)(bd^3 b_1 d_1^3)}{3(bd^2 - b_1 d_1^2)} - \dfrac{2(bd^2 b_1 d_1^2)^2}{}$	$\sqrt{\dfrac{bd^3 - b_1 d_1^3}{3(bd - b_1 d_1)} - \dfrac{(bd^2 - b_1 d_1^2)^2}{(2(bd - b_1 d_1))^2}}$
(channel section)	$bd - b_1 d_1$	$\dfrac{d}{2}$	$\dfrac{bd^3 - b_1 d_1^3}{12}$	$\dfrac{bd^3 - b_1 d_1^3}{6d}$	$\sqrt{\dfrac{bd^3 - b_1 d_1^3}{12(bd - b_1 d_1)}}$

APPENDIX II

TORSION TEST

IT was shown in Chapter IX that when a member is subjected to a twisting moment the material is shear stressed. In order to find the relationship between shear stress and strain, and to find the modulus of rigidity of the material a torsion test is carried out.

A suitable apparatus is shown in Fig. 144. The ends of the specimen S are enlarged and either square to fit the grips G_1 and G_2, or are circular and keyed to G_1 and G_2. By moving the jockey weight W along the lever arm L a torque is applied to the specimen at G_1, G_2 being fixed. The resultant twist of the specimen will make the lever drop; it is brought back to the horrizontal by turning the grip G_2 by means of the worm and worm-
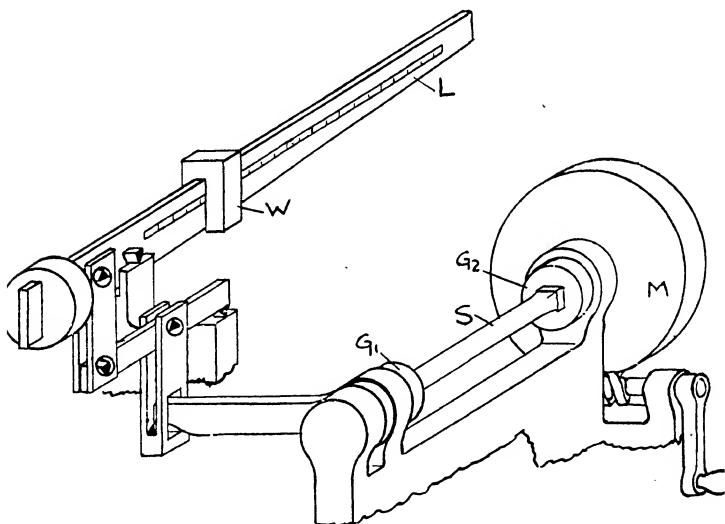
FIG. 144

wheel M. The torque may be read from a scale on the lever L, which is usually graduated to read torque in inch-lb.

The angle of twist on a given length is measured by a torsion meter, which may be similar to that shown diagrammatically in Fig. 144A. The torsion meter grips the specimen by the set screws (s). On the specimen twisting part a turns relative to part b, the angle of twist for the length l being read from the graduated scale g and the vernier v, viewed through microscope m.

A ductile material will fracture across a plane at right angles to the axis of the specimen, after making two or three complete revolutions. With a brittle material the angle of twist at fracture is small, and the fracture is in the form of a helix of about 45°. This is due to the material being able to resist a larger shearing than tensile stress. The specimen fails in tension.

It was shown in Chapter V that every shear force has a balanc-

ing shear force at right angles, and also that the force may be resolved into rectangular components. Consider a small square portion of the specimen $ABCD$, Fig. 145. The shear force P will be equal to the balancing shear force Q. Resolve these forces in a direction at right angles to BD, this gives a tensile force T

FIG. 144A

LINE OF FRACTURE
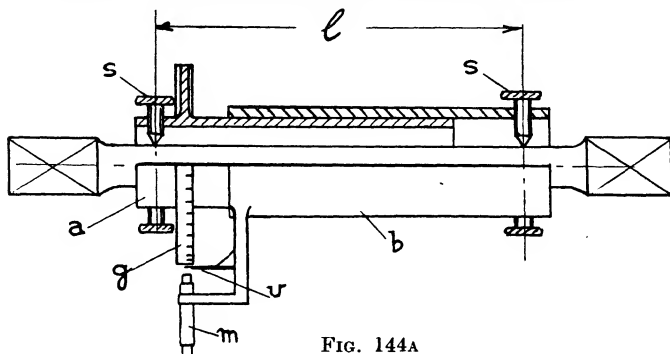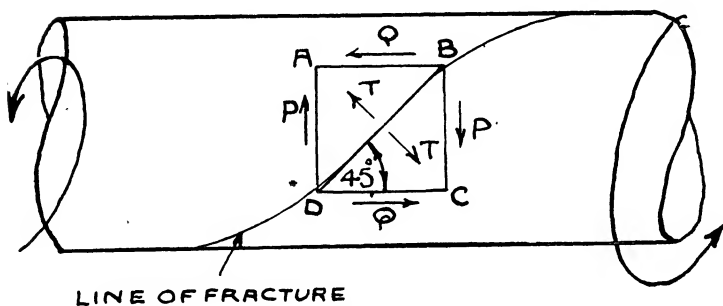
FIG. 145

along BD equal to $2P \cos 45°$. Let the area on which P and Q act be A, then the area on which T acts is $2A \cos 45°$.

Thus shear stress along $AD = P/A$

and tensile stress along $BD = \dfrac{2P \cos 45}{2A \cos 45} = \dfrac{P}{A}$

i.e. tensile stress at 45° to the axis is equal to the shear stress at right angles to the axis. Thus a material weaker in shear will fail by shear stress along AD, but a material weaker in tension will fail by tensile stress along BD.

TEST RESULTS

From the results of the test a shear stress-strain graph may be plotted.

Max. shear stress $= \dfrac{T16}{\pi D^3}$ (for solid specimen).

Max. shear strain $= \dfrac{\pi D \theta}{360 l}$ radians.

Where θ = angle of twist, degrees.

and l = length under test.

The results of a test for a medium carbon steel are given in the following table and the stress-strain graph in Fig. 146. The specimen was solid 1 in. diam. and 8 in. long.

Torque in. lbs.	Stress lb./sq. in. = $T/\cdot196$	Angle of Twist Degrees	Strain = $\dfrac{\pi\theta}{360 \times}$
0	0	0	0
1,000	5,100	0·395	·00043
2,000	10,200	0·770	·00084
3,000	15,300	1·165	·00127
4,000	20,400	1·53	·00167
4,600	23,500	1·74	·0019
4,700	24,000	13·0	·0142
6,000	30,600	18·4	·020
8,000	40,800	43·1	·047
10,000	51,000	84·5	·092
12,000	61,200	179·0	·195
14,000	71,500	339·5	·37
16,000	83,600	697·0	·76
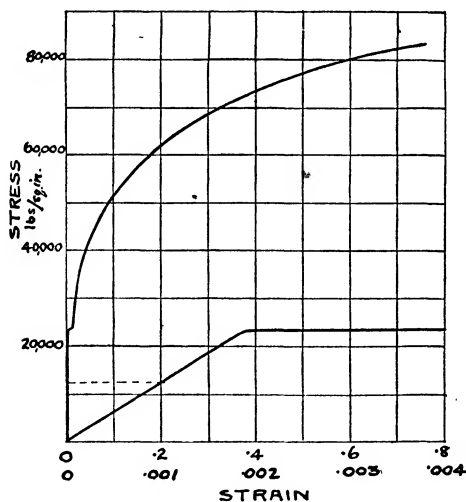
Fɪɢ. 146

It will be seen that the graph is of a similar form to a tensile stress-strain graph. There is a proportional stage, a yield, and a ductile portion.

The results show an ultimate shear stress = 83,400 lb./sq. in.
yield stress = 24,000 lb./sq. in.

The modulus of rigidity found from the straight portion of the large scale graph equals

$$\frac{12,100}{\cdot001} = 12,100,000 \text{ lb./sq. in.}$$

EXAMPLES

1. (*a*) What is the difference between the solid, liquid and gaseous states of a substance ?

(*b*) What takes place in the internal structure of a solid when it is melted ?

2. What is a crystal, and how is it formed ?

3. By what means is the structure of a metal revealed for examination under the microscope ? What appearance does a pure metal present ?

4. What is an alloy ? State three ways in which it may solidify.

5. What do you understand by—
    (*a*) A solid solution ?
    (*b*) A eutectic ?

6. Sketch typical cooling curves for the following—
    (*a*) Pure metal,
    (*b*) Alloy forming a solid solution,
    (*c*) Eutectiferous alloy.
Explain concisely the reasons for the shape of the curves.
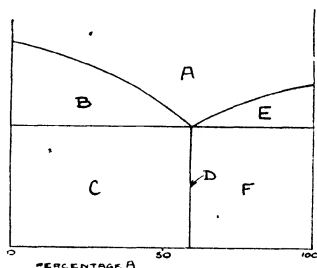
Ex. 7

7. How is a constitutional diagram constructed ?

The sketch shows an ideal constitutional diagram for alloys of two metals, *A* and *B*. What is the state of the mixture at points *A*, *B*, *C*, *D*, *E* and *F* ?

8. Sketch a cooling curve for pure iron, and state what takes place at each arrest point.

9. Make a sketch of the portion of the iron-carbon ·constitutional diagram that deals with steels. Show on the different spaces of the diagram the micro-constituents that exist under the conditions represented there.

10. What is—
    (*a*) Graphite ?
    (*b*) Pearlite ?
    (*c*) Cementite ?
    (*d*) Austenite ?

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11. What changes take place in the cooling from the liquid state of a hyper-eutectoid steel ? Include in your answer the names of the critical points at which these changes take place.

12. Explain clearly why low carbon steels are unsuitable for heat treatment.

13. What are the principal characteristics of—

     (*a*) Martensite ?

     (*b*) Sorbite ?

14. What is the object of annealing steel, and how is the process carried out when both strength and ductility are required ?

15. How is a case-hardened steel produced, and what are the advantages of such a steel ?

16. Discuss the chief properties of the following alloy steels—

     (*a*) Nickel steel,

     (*b*) 13 per cent. Chromium Steel,

     (*c*) Nickel-Chromium Steel.

17. What are the advantages and disadvantages of duralumin for use in aircraft structures ?
What is the effect of normalizing this alloy ?

18. Discuss the precautions taken by the aeronautical engineer against corrosion of steels and light alloys.

19. Define : Stress, Strain, Elasticity.
A flat bar $1\frac{1}{2}$ inches wide, $\frac{1}{4}$-inch thick and 4 feet long is secured by one $\frac{1}{2}$-inch diameter bolt at each end. What is the maximum tensile stress in the bar when it is subjected to a tensile load of 4,000 lb. ? Find also the amount the bar elongates under this load if Young's Modulus = 30,000,000 lb./sq. inch.

     Ans. : 16,000 lb./sq. inch.

          0·017 inch.

20. If there is a tension of 1 ton in the member shown in the sketch find—

     (*a*) The maximum tensile stress in *A*,

     (*b*) The maximum tensile stress in *B*,

     (*c*) The shear stress in the rivets,

     (*d*) The maximum bearing stress.
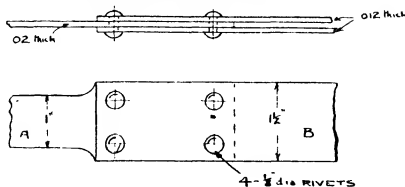
Ex. 20

  Ans. : (*a*) 50 tons/sq. inch.

        (*b*) $33\frac{1}{3}$ tons/sq. inch.

        (*c*) 10·2 tons/sq. inch.

        (*d*) 100 tons/sq. inch.

21. State Hooke's Law.

What load in pounds must be hung on an iron wire 50 feet long and 0·1 inch diameter to make it stretch $\frac{1}{5000}$ inch ? E=28,700,000 lb/sq. in.

Ans. :   0·0752 lb.

22. The maximum allowable stress in the material of the tube in the sketch is 20 tons/sq. inch. It is subjected to a tensile load of 7,500 lb. If the thickness of the material is 0·025 inch, find a suitable external diameter, and the shear stress in the taper pins.

2-¼" dia TAPER PINS

2-¼" dia TAPER PINS
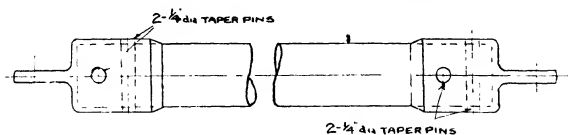
Ex. 22

Ans. : 2·3 inches.   17 tons/sq. inch.

23. Define Factor of Safety.

A tie bar has to sustain a pull of 7·5 tons. Calculate the necessary diameter of the bar if the ultimate tensile strength of the material is 32 tons/sq. inch, and a factor of safety of 5 is adopted.

Ans. : 1·22 inches.

24. Define Load Factor.

The forked end of a flying-wire fitting is connected to a lug on the main spar by a pin such that it is in double shear. Find the necessary diameter of the pin using a yield stress of 40 tons/sq. inch, and a proof factor of 5. The load in the flying wire in steady flight is 3,000 lb.

Ans. : $\frac{3}{8}$ inch diameter.

25. Two flat steel bars are to be fastened together by a bolt of diameter d inches, to form a tension member. The bars are w inch wide and their resistance to tearing is to equal the bolt's resistance to shearing. Find a formula for the thickness ' t ' of the bars.

Let f_s = maximum shear stress and f_t = maximum tensile stress.

Ans. : $t = \dfrac{f_s \pi d^2}{f_t 4(w-d)}$ inch.

26. What do you understand by Poisson's Ratio ?

A steel rod 2-inch diameter passes through a ring such that there is 0·001 inch clearance when there is no load in the rod. What will be the clearance when the rod is subjected to a tensile load of 60 tons ? Take $m=4$.

Ans. : 0·00175 inch.

27. Define Modulus of Rigidity.

Two bars are lapped and riveted together to form a tensile member. Four ¼-inch diameter mild steel rivets are used. Find the shear strain when there is a pull of 6,280 lb. in the bars, given modulus of rigidity equals 12,000,000 lb./sq. inch.

$$\text{Ans.} : \frac{1}{375}\text{radians.}$$

28. Define the following terms—

        (a) Ductility.   (d) Elastic Limit.

        (b) Hardness.   (e) Ultimate Strength.

        (c) Proof Stress.

29. Write what you know about the phenomenon of strain in a ductile material.

30. During a tensile test on a standard test piece of duralumin the following results were obtained—

Load, tons ...	0·5	1·0	2·0	3·0	4·0	4·5	5·0	5·5	6·0
Extension, ins.	0·0008	0·0016	0·0034	0·0051	0·0070	0·0080	0·030	0·058	0·092

Maximum load required to break specimen   ...   7·5 tons.

Gauge length ...   .   2 inches.

Distance between gauge points after fracture   ...   2·24 inches.

Diameter of test piece   ...   .   0·564 inch.

(a) Plot stress-strain graphs both for elastic straining and to fracture and obtain from them the elastic limit and Young's Modulus.

(b) Find the percentage elongation, and state whether or no there will be a big reduction of area at the point of fracture. Give reasons for your answer.

        Ans. : (a) 15·5 tons/sq. inch.   4,800 tons/sq. inch.

              (b) 12 per cent.   No.

31. Sketch an approximate stress-strain graph for mild steel and show on it the effect of removing the stress at some point between yield and ultimate stress. Mark on the graph the yield stress of—

        (a) The original specimen,

        (b) The overstrained specimen.

32. How is the ductility of a metal tested—

        (a) In the case of bar ?

        (b) In the case of sheet metal ?

33. What precautions must be taken to ensure consistent results in the Brinell hardness test ?

What approximate relationship does the hardness number bear to ultimate strength of

        (a) Steels ?

        (b) Light alloys ?

34. What information may be obtained from the Izod value of a material ? How would you carry out a test to obtain this value ?

35. State briefly what is meant by the fatigue of metals, and give three practical examples.

How does the surface finish and corrosion affect a metal's resistance to fatigue failure ?

36. Sketch the approximate S/N curve you would expect to obtain from a standard rotating cantilever test, using a solid specimen of steel which has an ultimate stress of 36 tons/sq. inch.

Mark on your graph the approximate endurance limit and state why this may be taken on the basis of 10 million reversals.

37. In a Wohler fatigue test of a specimen of forged " Hiduminium " Y alloy the following results were obtained—

Max. Stress, tons/sq. in.	13·5	11·5	10·5	10	9·5	9·4	9·35	9·32	9·31
Reversals, millions	0·4	1·9	7·05	12·4	23·2	27·1	29·6	36·0	50

Plot the S/N graph, and give what you consider to be the endurance limit.

Ans. : 9·3 tons/sq. inch.

38. Show with the aid of sketches the ways in which a riveted joint may fail.

A load of 3,200 lb. is transmitted to a metal spar by means of two 0·036 inch thick side plates riveted to the spar flanges, such that the rivets are in single shear.

The flanges are 0·022 inch thick and solid $\frac{3}{32}$-inch diameter rivets, are used. If the maximum allowable shear and bearing stresses are 18 tons/sq. inch and 36 tons/sq. inch respectively, find the number of rivets.

Ans. : 20.

39. Obtain a formula for the pitch of rivets in a treble-riveted lap joint.

Ans. : $\dfrac{f_s 3\pi d^2}{f_t 4t} + d.$

40. Find the pitch of rivets in, and the efficiency of, a single-riveted butt joint, if the plate is $\frac{1}{2}$ inch thick and $\frac{3}{4}$ inch diameter rivets are used. Take the allowable shear stress of the rivets to equal 0·8, the allowable tensile stress of the plate.

Ans. : 2·16 inches.   65 per cent.

41. A piece of rib tension bracing is repaired as shown in the sketch. All the material is the same in thickness and strength. Is the repair 100 per cent. efficient if the rivets A are—
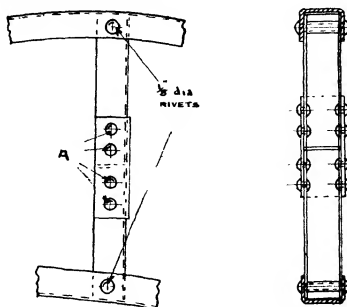
Ex. 41

(a) $\frac{3}{32}$ inch diameter ?

(b) $\frac{5}{32}$ inch diameter ?

Give the reasons for your answers.

Ans. : (a) Yes. (b) No.

42. The sketch shows diagrammatically the arrangement for a control pulley attachment to a spar. If the pull in the cable is 400 lb. and the maximum permissible shear and bearing stress are 16 tons/sq. inch and 40 tons/sq. inch respectively, find—

(a) The direct load in the rivets of plate A.

(b) The direct load in the rivets of plate B.

(c) The load due to Torque in rivets 1, 2 and 3 of plate A.

(d) The size of rivets required if all the rivets are to be the same size.
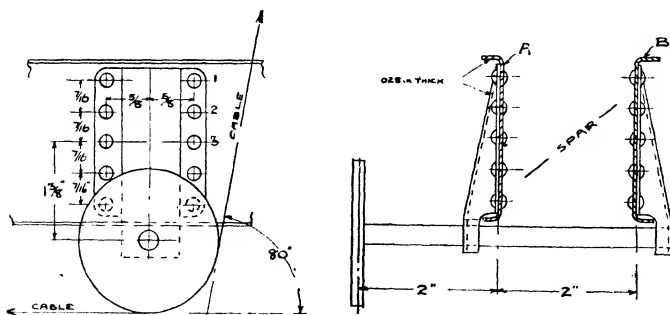
Ex. 42

Ans. : (a) 103 lb. each.

(b) $51\frac{1}{2}$ lb. each.

(c) 119·3 lb., 84·7 lb., 69·4 lb.

(d) $\frac{3}{32}$ inch.

43. Design a wiring lug of uniform strength for a $\frac{3}{8}$-inch B.S.F. streamline wire, which transmits a load of 8,500 lb. The standard pin to be used is $1\frac{3}{32}$ inch diameter, and the fork end is such that the distance from the pin centre to end of lug must not exceed $\frac{11}{16}$ inch, and width of jaw is 0·17 inch. Permissible tensile stress in the material, 45 tons/sq. inch, and bearing stress 90 tons/sq. inch.

Ans.: $t=0\cdot128$ inch, $W=0\cdot66$ inch and $R_2=0\cdot67$ inch.

44. A 4-inch internal diameter pipe is subjected to an internal pressure of 480 lb./sq. inch. The ends are closed by means of a cap screwed to the pipe, as in the sketch.
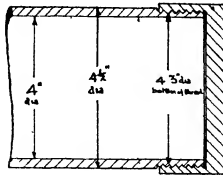
Ex. 44

Find (a) the hoop stress,

(b) The maximum longitudinal stress.

Ans.: (a) 3,840 lb./sq. inch.

(b) 3,200 lb./sq. inch.

45. An aeroplane spar has a $\frac{1}{8}$-inch diameter rivet hole punched in it. If the material is 0·04 inch thick and has an ultimate shear strength of 50 tons/sq. inch, what force is required?

Ans.: 1,760 lb.

46. A symmetrical block weighing 3,000 lb. is supported, as in the sketch, by three wires of equal diameter and length. The centre is a steel wire, and the two outer wires are brass. If Young's Modulus for steel is 13,000 tons/sq. inch, and for brass 6,000 tons/sq. inch, find the load taken by each wire. Any deflection of the block to be neglected.
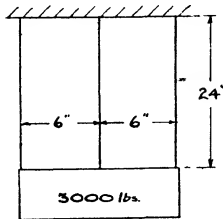
Ex. 46

Ans.: Brass, 720 lb.
Steel, 1,560 lb.

47. Duplicate lift wires of equal length are each 0·038 sq. inch cross-sectional area. One wire is shot away when a load of 7,600 lb. is being transmitted ; what is the load and stress in the remaining wire at the moment of breaking ?

Ans. : 11,400 lb.   134 tons/sq. inch.

48. A 1-foot-high column, 8 inches diameter outside and 6 inches diameter inside, carries a load of 88 tons. Find the normal and tangential stresses on a section whose plane makes an angle of 30° with the normal section.

Ans. : 3 and 1·732 tons/sq. inch.

49. What diameter of solid shaft will be required to transmit 80 h.p. at 60 r.p.m. if the maximum permissible shear stress is 8,000 lb./sq. inch ?

The torque varies throughout each revolution, and the maximum torque is 130 per cent. of the mean torque.

Ans. : 4·1 inches.

50. Explain clearly with the aid of sketches why a hollow shaft gives a greater strength/weight ratio than a solid shaft.

51. A motor is coupled up to a machine by means of a solid shaft with a flanged coupling, the latter having 4 bolts, each $\frac{3}{8}$-inch diameter, on a pitch circle of 1·25-inch radius. What must be the minimum diameter of the shaft, so that it will not fail before the coupling bolts, if the ultimate shear stress of the shaft and bolts is the same ?

Ans. : 1·41 inches.

52. A hollow shaft, 2-inch outside diameter and 1½-inch inside diameter, is 8 inches long and twists through an angle of 1·6° under a torque of 400 ft. lb. Find the stress in the shaft and the modulus of rigidity of the material.

Ans. : 4,470 lb./sq. inch.
       4,023,000 lb./sq. inch.

53. A solid shaft 2½-inch diameter is subjected to a torque, and the resulting maximum shear stress is 5,000 lb./sq. inch. If the diameter of the shaft is reduced to 2 inches, and the same torque applied, what is now the shear stress in the material ?

Ans. : 9,770 lb./sq. inch.

54. A close-coiled steel spring is required to extend 1 inch for every 20 lb. load. If the coils are to have a mean radius of ½ inch, and the diameter of the wire is 0·128 inch, find the number of coils. $C=$ 12,000,000 lb./sq. inch.

Ans. : 20.

55. By means of diagrams show the distribution of stress due to the pure bending of a beam—

(*a*) Of I section.

(*b*) Of T section.

What is meant by—

(*c*) The Neutral Axis ?

(*d*) The Moment of Resistance ?

56. A screwed bolt is required to carry a pull of two tons. The maximum stress allowed is 3 tons/sq. inch. If the diameter at the bottom of the thread is nine-tenths of the diameter of the bolt, find the size of the bolt required.

Ans. : 1 inch.

57. A beam is freely supported at *A* and *B*, and loaded as shown in the sketch. Draw the bending moment and shearing force diagrams, neglecting the weight of the beam. Find also the distance of the point of inflection from *A*.
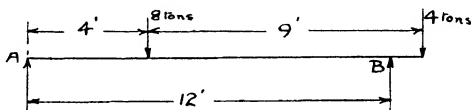
Ex. 57

Ans. : $10\frac{2}{3}$ feet.

58. A cantilever of uniform section carries a vertically upward load of 20 lb. per foot, and weighs 4 lb. per foot. If it is 20 feet long and has a modulus of section of 8·4 inches³, find the maximum stress in the material.

Ans. : 4,571 lb./sq. inch.

59. Design a close-coiled helical spring of round steel wire to carry a total load of 400 lb. with a compression of 4 inches. Maximum stress allowed 16 tons /sq. inch. $C=12,000,000$ lb./sq. inch.

60. A steel shaft is required to carry 40 h.p. at 150 r.p.m. The maximum torque is twice the mean, and the shear stress is limited to four tons per sq. inch.

What diameter of shaft is necessary ?

If a square part has to be cut on the shaft, how much should the diameter be increased ?

Ans. : 2·1 inches. 0·7 inch.

61. Two men carry a uniform plank freely supported by the ends into a shed, where they saw it in half. They return with one half carried in the same way as before. If the deflection of the whole plank on the inward journey was 1½ inches, what was the deflection of the half plank on the return journey ?

Ans. : $\frac{3}{32}$-inch.

62. In the sketch, A is a square-section cantilever, and B is a round bar pinned to A by the pin C. and supporting a weight of 2,000 lb.

Find the minimum sizes of A, B and C, if the stress in each case is not to exceed 12 tons/sq. inch.
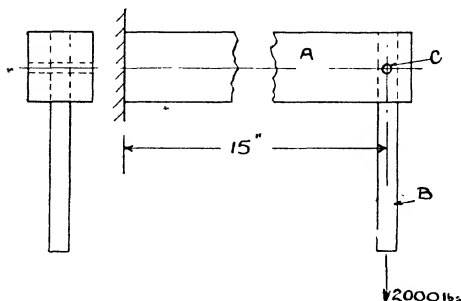
Ex. 62

Ans. : A, 1·9-inch sides.

B, $\frac{1}{2}$-inch diameter.

C, $\frac{7}{32}$-inch diameter.

63. AB is a 4-foot length of beam. At A the bending moment is 1,200 ft. lb. If the shearing force increases uniformly from 80 lb. at A to 120 lb. at B, what is the bending moment at B ?

Ans. : 1,600 ft. lb.

64. In a test on a rotating cantilever fatigue-testing machine using the standard solid specimen, the following results were obtained—

Load, lb.	120	105	95	85	79	78	77
Endurance, millions ...	0·06	0·2	0·6	1·3	5·5	9·6	11·5*

* Unbroken.

Plot the S/N graph, and find from it the endurance limit.

Ans. : 37,000 lb./sq. inch.

65. On what does the strength of a beam depend ?

Explain with the help of sketches why an I section beam gives a relatively high strength/weight ratio.

66. The ratio of the stress at any point in a simple beam, to the distance of that point from the neutral axis, is equal to the ratio of the modulus of elasticity of the material to the radius of curvature at the neutral axis. Prove this statement.

67. A beam 14 feet long is freely supported at one end, and 4 feet from the other end. It carries a uniformly distributed load of 100 lb. per foot over the whole length, and a concentrated load of 2 tons at a distance of 4 feet from the end at which there is a support. Draw the shearing force and bending moment diagrams.

68. If the beam in Example 67 is of uniform rectangular section and 5 inches deep, what will be the minimum breadth if the maximum permissible stress is 10 tons/sq. inch ? Find also the maximum shear stress.

Ans. : 1·5 inches. 622 lb./sq. inch.

69. A 9-foot beam is freely supported at each end, and carries a load increasing uniformly from 20 lb./ft. at one end to 50 lb./ft. at the other end. Draw the shearing force and bending moment diagrams, and write down the maximum shearing force and bending moment.

Ans. : 180 lb. 357 ft. lb.

70. A 20-feet long steel beam of I section is 12 inches deep, the flanges are 4 inches wide, and the metal is ½ inch thick. It is freely supported at the ends, and loaded uniformly until the maximum stress in the metal is 7 tons/sq. inch. Calculate the deflection at the centre. Take E=13,000 tons/sq. inch.

Ans. : 0·538 inch.

71. A solid 2-inch diameter shaft for transmitting pure torsion is made hollow by having a ½-inch diameter hole drilled through the centre. What will be the percentage increase in strength/weight ratio ?

Ans. : 6¼ per cent.

72. Obtain a formula for the deflection at the free end of a cantilever, which carries one concentrated load at the centre of its length.

Ans. : $\dfrac{5Wl^3}{48\,EI}$

73. The maximum deflection of a certain steel beam is found to be ½ inch. What will be the maximum deflection of an exactly similar duralumin beam, carrying half the load. E for steel=13,000 tons/ sq. inch. E for duralumin =4,700 tons/sq. inch.

Ans. : 0·69 inch.

74. A steel tubular beam freely supported at the ends is 2 inches outside diameter, 0·056 inch thick and 3 feet long. It carries an evenly distributed load of 40 lb. per foot. What is its maximum deflection ?

Ans. : 0·015 inch.

75. A beam fixed at one end rests freely on a support at the other end, such that the two ends are level. If the beam is loaded with a concentrated load W at the centre, find the reaction P at the support in terms of W.

Ans. : $P = \frac{5}{16} W$.

76. The beam ABC is symmetrically loaded as shown in the sketch. Obtain a formula for the deflection upwards at the centre B above the supports.
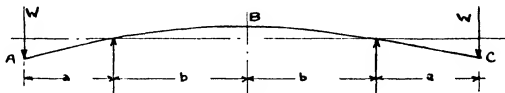
Ex. 76

Ans. : $\dfrac{Wab^2}{2\,EI}$

77. A beam 120 feet long rests on four supports, the three spans being 30, 50 and 40 feet respectively. If the beam is carrying a uniformly distributed load of 120 lb./ft. over the whole length, find the maximum bending moment.

Ans. : 25,800 ft. lb.

78. A steel beam 70 feet long weighs 60 lb. per foot and rests on three supports A, B and C. Support A is at one end, support B 30 feet from A, and support C at the other end. Span AB carries a concentrated load of 2,000 lb. 20 feet from A, and span BC carries an evenly distributed load of 50 lb./ft. for its whole length. Find the reactions at A, B and C.

Ans. : 892 lb. ; 5,614 lb. ; 1,694 lb.

79. A 2-inch diameter, 5-inch long bar is fixed horizontally at one end, and by means of an arm carries a vertical load of 120 lb. at a horizontal distance of 4 inches from the axis at the free end. Find the equivalent torque and the maximum stress in the material under this condition.

Ans. : 768 in. lb., 490 lb./sq. inch.

80. A mild steel tube, 2-inch diameter outside and 0·036 inch thick, is used as a strut 5 feet long with fixed ends. The maximum stress allowed is 5 tons/sq. inch. What load may it carry ? Use Rankin's formula.

Ans. : 1,250 lb.

81. What is the effect on the maximum bending moment of a beam of (a) a tensile end load, (b) a compressive end load ?

In case (b) would you choose a material with a high or low modulus of elasticity ? Give reasons for your answers.

82. What diameter of 0·025-inch thick steel tube is required to carry a compressive load of 250 lb., if the strut is to be 12 feet long ?

Ans. : 1¼ inch.

83. Construct a graph showing the variation of buckling load per unit area, with slenderness ratio, for struts made of grade A spruce. Take ultimate strength = 5,000 lb./sq. inch, and E = 1,600,000 lb./sq. inch.

84. What are the limitations of—

(*a*) Euler's Formula,

(*b*) Rankin's Formula ?

What is the advantage of corrugating a thin metal strut ? Would any advantage be gained by corrugating a tubular strut 1½-inch outside diameter and 0·048-inch thick ?

85. At a point in the cross-section of a beam there is a tensile stress of 4 tons/sq. inch normal to the cross-section, and a shear stress of 2 tons/sq. inch. Find the principal stresses and the direction of the principal planes to the cross-section.

Ans. : 4·828 at 22½° tension.
0·828 at 67½° compression.

86. A rivet 1 inch diameter is under a pull of 4 tons and a shear force of 3 tons. Find the direction and magnitude of the principal stresses.

Ans. : 7·15 ton tension 61·8°.
2·05 ton compression 28·2°.

87. A short column 8 inches outside diameter and 6 inches inside diameter carries a load of 88 tons. Find the normal and tangential stresses on a plane section at 30° to the normal section.

Ans. : 3, $\sqrt{3}$

88. A cantilever 15 feet long carries a uniform loading of 80 lb./ft. run. Find the deflection and slope at the end, if the Moment of Inertia cross-section = 18·4 in.⁴ and E = 30 × 10⁶ lb./sq. inch.

Ans. : 1·59 inches, ·0118 radians.

89. A beam 12 feet long and simply supported at its ends carries an evenly distributed load of ½ ton per foot over the whole span, and a compressive end load of 10 tons. Given that the resultant deflection at the centre is 0·84 inches, find the maximum stress. The beam is rectangular in cross-section, 6 inches deep and 2 inches wide.

Ans. : 10·53 ton/sq. inch.

90. A Wagner beam, 10 feet long, is freely supported at its ends, and carries a uniformly distributed load of 400 lb./ft. Each flange has a cross-sectional area of 0·12 sq. inch, and the webs are 10 inches deep and 0·022 inch thick. Find the maximum stress in the web and tensile stress in the flange.

Ans. : 18,200 lb./sq. inch.
41,700 lb./sq. inch.

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